Supplementary Appendix (Not for Publication) to 'Full Discretion is Inevitable'

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One critical assumption for Theorem 1 is that each singleton type has positive probability mass in the prior—$\mu_0(\{p\}) > 0$ for each $p \in N$. If the Principal and Agent have perfectly aligned preferences (π_p is also increasing in p), the Principal has no incentives to restrict the project choice of the Agent, hence Theorem 1 will hold even when this assumption is violated. In contrast, if the Principal and the Agent have opposite preferences over project choice, there exist many other equilibria with fundamentally different equilibrium outcomes whenever it is common knowledge that the Agent has at least two feasible projects:

Proposition 1. *Suppose the Principal and the Agent have opposite preferences over project choice (π_p is strictly decreasing in p), and it is common knowledge that the Agent has at least two feasible projects ($|S| \geq 2$ for each type S in support of the prior μ_0),*

> *(i) When the Agent is sufficiently patient—, $\delta_A > \bar{\delta}$, for each project p, there exist a corresponding stationary equilibrium where the Principal permits all but project p at every history;*
>
> *(ii) When the Agent is sufficiently patient—, $\delta_A > \bar{\delta}$, there exists a trigger strategy equilibrium where the Principal's payoff converges to the First Best Payoff as δ_P goes to one.*

(iii) Suppose the worst project for the Principal gives her zero payoff and is always feasible ($\pi_n = 0$ and $n \in S$ for each S in support of the prior). When the Agent is sufficiently patient—, $\delta_A > \bar{\delta}$, any strategy of the Principal can be supported in equilibrium. In other words, it is without loss to assume the Principal can commit to future actions.

Proof. **Part (i):** Take any project p. To construct the corresponding stationary equilibrium, let the Agent choose the following strategy: type S with $\max S \neq p$ follows the NC strategy; type S with $\max S = p$ first considers project p and then considers project $\max S \setminus \{p\}$ and accepts no other project. In other words, Agent of such type S choose p if it is permitted; if p is not permitted but $\max S \setminus \{p\}$ is permitted, chooses $\max S \setminus \{p\}$; otherwise, chooses 0.

It is straightforward to verify that the Principal's strategy and the Agent's strategy (coupled with a belief derived by Bayes' rule from the prior and the Agent's strategy) constitute an equilibrium: Because the Agent is patient—$\delta_A > \bar{\delta}$ and the Principal permits every project other than p next period, type S with $\max S \neq p$ will not make any compromise; type S with $\max S = p$ will first consider p and then consider $\max S \setminus \{p\}$ and will not make any further compromise. Hence the Agent will not deviate. For the Principal, because only type S with $\max S = p$ is willing to make compromises and accept the second favorite feasible project, the best the Principal can do is to force each such type S to choose $\max S \setminus \{p\}$. Proposing $N \setminus \{p\}$ achieves exactly that. Hence, the Principal also does not have any strictly profitable deviation.

Part (ii): Let K be the index of the best feasible project for the Principal from the most "pessimistic" type:

$$K = \max\{k | \min S = k \text{ for some type } S \text{ with } \mu_0(S) > 0\}.$$

Because each type has at least two feasible projects, I have $K \leq n-1$. Let T be a sufficiently large number such that no Agent type is willing to wait T periods to obtain a better project.

$$T \equiv \min\{t | \alpha_n \delta_A^t < \alpha_1\}.$$

On the equilibrium path, the Principal proposes A_t in period t:

$$A_t \equiv \begin{cases} \{1, \ldots, k\} & T \cdot (k-1) \leq t \leq T \cdot k - 1 \text{ for } k = 1, \ldots, K-1 \\ \{1, \ldots, K\} & t \geq T \cdot (K-1), \end{cases}$$

For each $k = 1, \ldots, K$, denote the set of Agent types with k as the best (worst) feasible project for the Principal (the Agent) by $\mathcal{S}_k$:

$$\mathcal{S}_k \equiv \{S | \min S = k\}.$$

$\{\mathcal{S}_k\}_{k=1}^{k=K}$ forms a partition of the support of the prior.

Each type $S \in \mathcal{S}_k$ has no feasible project permitted before period $T \cdot (k-1)$, and in period $T \cdot (k-1)$, type S has no incentive to wait for projects better than k.

The trigger strategy equilibrium is constructed as follows: In each period t, if the Principal proposed A_τ in each past period $\tau < t$, she continues to propose A_t;

otherwise, he follows FD strategy by proposing N. For the Agent, if the Principal proposes equilibrium permission sets $\{A_\tau\}_{\tau=0}^{\tau=t}$ in all past and the current period, he chooses optimal action given that current permission set is A_t and future permission set is A_j in each period $j > t$; if the Principal ever deviates from the equilibrium permission set, the Agent switches to follow NC strategy.

A sufficient condition for the above to constitute an equilibrium is that the Principal has incentives to propose A_t in period t given that she has proposed A_τ in each past period $\tau < t$. In period 0, the Principal has the following equilibrium payoff:

$$\sum_{k=1}^{k=K} \delta_P^{T\cdot(k-1)} \sum_{S\in\mathcal{S}_k} \mu_0(S)\pi_k.$$

Because the Agent responds to any deviation by the NC strategy, the best deviation for the Principal is to propose N which yields the following payoff:

$$\sum_{k=1}^{k=K} \sum_{S\in\mathcal{S}_k} \mu_0(S)\pi_{\max S}.$$

For each $k = 2, \ldots, K$, in each period t with $T \cdot (k-2) < t \leq T \cdot (k-1)$, equilibrium continuation payoff is:

$$\sum_{j=k}^{j=K} \delta_P^{T\cdot(j-1)-t} \sum_{S\in\mathcal{S}_j} \mu_0(S)\pi_j.$$

Again, the best deviation for the Principal is to deviate to propose N, which yields the following payoff:

$$\sum_{j=k}^{j=K} \sum_{S\in\mathcal{S}_j} \mu_0(S)\pi_{\max S}.$$

For $t > T \cdot (K-1)$, the information set is necessarily off the equilibrium path because all Agent types should have left the game, I can specify the Principal's posterior belief so that all types in support of her belief are in set $\mathcal{S}_K$. By sticking to the equilibrium permission set $\{1, \ldots, K\}$, the Principal receives her favorite feasible project from each possible Agent type, hence has no incentive to deviate.

As a result, for the above strategy profile to constitute an equilibrium, it is sufficient to let

$$\sum_{j=k}^{j=K} \delta_P^{T\cdot(K-1)} \sum_{S \in \mathcal{S}_j} \mu_0(S)\pi_j \geq \sum_{j=k}^{j=K} \sum_{S \in \mathcal{S}_j} \mu_0(S)\pi_{\max S},$$

for each $k = 1, \ldots, K$.

Equivalently,

$$\delta_P^{T\cdot(K-1)} \geq \max_k \frac{\sum_{j=k}^{j=K} \sum_{S \in \mathcal{S}_j} \mu_0(S)\pi_{\max S}}{\sum_{j=k}^{j=K} \sum_{S \in \mathcal{S}_j} \mu_0(S)\pi_j}$$

Because T does not depend on δ_P, and $\pi_j > \pi_{\max S}$ for each type $S \in \mathcal{S}_j$, I can find δ_P sufficiently close to one that this condition is satisfied.

Under this equilibrium, the Principal obtains equilibrium payoff:

$$\sum_{k=1}^{k=K} \delta_P^{T\cdot(k-1)} \sum_{S \in \mathcal{S}_k} \mu_0(S)\pi_k,$$

which converges to $\overline{V}$ as δ_P goes to one.

Part (iii) has a very simple logic: Given any strategy of the Principal, to support it as part of an equilibrium, whenever the Principal deviates from this strategy in the past, both players immediately shift to play according to full discretion equilibrium (FD, NC), which gives the Principal zero payoff. Therefore, any deviation of the Principal leads to zero payoff and cannot be profitable.

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