Full Discretion is Inevitable*

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Abstract

This paper studies a dynamic project-selection game between a Principal and an Agent with conflicting interests. Only the Agent knows what projects are available. In each period before a project is selected, the Principal imposes a restriction set. The Agent can select any available project within this set, thereby ending the game. The Agent can also stay silent, in which case the game will proceed to the next period. Importantly, the Principal cannot commit to her future restriction sets. I show that when the Agent is sufficiently patient, the Principal fully delegates to the Agent in the unique equilibrium.

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1 Introduction

Consider the following incentive problem: a Principal (she) and an Agent (he) would like to agree on a project but only the Agent knows what projects are available. This kind of "project-choice" problem is ubiquitous: it manifests in the relationship between a university and a department when determining whom to hire or between a firm's division and its headquarters when considering a potential investment. The Agent in these examples is better informed about the availability of each project than the Principal, but their preferences may not align. The Principal then has to decide to what degree she should delegate the choice to the Agent. Full delegation always results in the selection of a project whenever one is available but results in the Agent always choosing his favorite project. By contrast, restricting the Agent to some subset of projects may tilt the Agent to selecting a project preferred by the Principal but also increases the chance that no project is selected, resulting in the status quo outcome.

In their seminal work, Armstrong and Vickers (2010) study this project-choice problem as a one-shot interaction: a Principal commits to a "restriction" wherein the Agent can propose a project only in a given set. Such a proposal is then selected. If the Agent has no project to propose in that restriction, both parties obtain the status quo. An important idea that emerges from this analysis is that to counteract the Agent's bias, the Principal may exclude projects that reflect Pareto gains relative to the status quo. Partial delegation thus emerges as an optimal solution.

This paper revisits this setting but with dynamic considerations in mind. Suppose that the Agent does not recommend a project in the Principal's restriction. The Principal may then infer that the projects in her restriction are infeasible. She may wish to give the Agent another chance by adjusting the restriction. Anticipating this response, the Agent may then hold back on proposing projects. How well can the Principal do in this setting?

The main result of this paper answers this question. Informally stated, the answer is as follows:

> **Theorem**. *If the Agent is patient, the unique equilibrium outcome involves the Principal fully delegating to the Agent.*

The key idea is that if the Principal does not fully delegate to the Agent from the start, the Agent has an incentive to hold back on proposing projects in the restricted set so as to convince the Principal that such projects are unavailable. At some point, the Principal must capitulate, letting the Agent choose. If the Agent is sufficiently patient, he will wait until that happens. The Principal is then trapped and would have been better off by capitulating at the outset.

The strategic logic encapsulated above is reminiscent of the reputational arguments that feature in Kreps and Wilson (1982) and Milgrom and Roberts (1982) or the Coase Conjecture (Gul, Sonnenschein, and Wilson, 1986). I highlight some important distinctions. Relative to reputation models, all the players in my setting are strategic players; more critically, I use a distinct argument to show the uniqueness of equilibrium as my model has an infinite horizon. Relative to the Coase Conjecture, types in my model are not ordered, and hence there is no analog to single crossing or the skimming property being satisfied here.[1] Second, the actions in this model are discrete—i.e., to approve or reject a project—yet the aforementioned outcome emerges across all equilibria. By contrast, in seller-buyer bargaining with discrete prices, non-Coasian equilibria are possible.[2]

Existing papers rationalize delegation by highlighting the benefits to the Principal. Aghion and Tirole (1997) show delegation encourages the Agent to take initiative, while Dessein (2002) show delegating to an informed Agent is better for the Principal than communicating with the Agent. This paper complements the existing work by identifying a new and distinct force: the Principal is forced to fully delegate project choice to the Agent because she is uncertain about what the Agent can do and she cannot commit in advance to her future restrictions.

The rest of the paper is organized as follows. Section 2 introduces the setup. Section 3 proves the main result of this paper (i.e., full discretion is inevitable

[1]Indeed, in a different setting, Ali, Kartik, and Kleiner (2022) show that failures of standard single crossing may lead to equilibria that fail the skimming property and result in the commitment payoff being attained.

[2]Von der Fehr and Kühn (1995) discuss (in Section 7) the gap case of seller-buyer bargaining when the price space is discrete. When the buyer is sufficiently patient, for each price p smaller than the lowest value of the buyer, there exists a stationary equilibrium wherein the good is sold immediately at price p. There are also trigger-strategy equilibria that support high prices on the equilibrium path by threatening to revert to the lowest-price stationary equilibrium.

in equilibrium) and compares the equilibrium outcome to relevant commitment benchmarks. Section 4 discusses related literature. Appendix A provides proofs omitted from the main text.

2 Model

A Principal (she) and an Agent (he) jointly choose a project. The set of potential projects is denoted by $N \equiv \{1, \ldots, n\}$, and the set of all non-empty subsets of N is denoted by $\mathcal{N}$. If project p is selected, then the Principal's payoff is π_p and the Agent's payoff is α_p; if no project is selected, then each player obtains 0. I consider the generic case in which the Agent has strict preference over the set of projects, and I order projects so that α_p is strictly increasing in p. I further assume that for each p, π_p and α_p are in $(0, 1)$. In my setting these are without loss of generality, as the Principal has no incentive to permit projects where she obtains a negative payoff and the Agent has no incentive to choose projects where he obtains a negative payoff.

The Principal does not know which projects are available or feasible for the Agent; this is private information possessed by the Agent. The set of feasible projects, $S \subseteq N$, is drawn according to the prior, $\mu_0 \in \Delta(\mathcal{N})$. Since $\mathcal{N}$ excludes the empty set, this presumes that there is always at least one project that is feasible. For expositional convenience, I further assume that μ_0 has positive probability on any singleton: for every p, $\mu_0(\{p\}) > 0$. I refer to S as the Agent's *type*. Given a set of feasible projects, S, $\max S$ (resp. $\min S$) denotes the feasible project that has the highest (resp. lowest) label.

I model a dynamic delegation game. At every period $t = 0, 1, \ldots,$ until a project is selected:

1. The Principal proposes permission set $A_t \subseteq N$.

2. The Agent chooses from set $(A_t \cap S) \cup \{0\}$.[3] Here, action 0 represents that the Agent stays silent.

[3] I implicitly assume the Agent cannot choose non-feasible or non-permitted projects. The former is without loss in contexts where non-feasible projects incur losses to the Agent. Whether my main result is robust to allowing the Agent to choose non-permitted projects is a question that I leave to future exploration.

- If the Agent chooses project p in $A_t \cap S$, the game ends with project p selected. I refer to this case as reaching an **agreement** on project p.
- If the Agent chooses 0, the game continues to period $t + 1$.

In this game a history is a sequence of permission sets. A strategy for the Principal is a function that assigns to every history a probability distribution over 2^N, interpreted as the (possibly random) permission set the Principal proposes given that the Agent has chosen 0 in all past periods. A strategy for the Agent is a function that—for each history, each type S, and each current permission set A—specifies the probability the Agent chooses each action in set $(A \cap S) \cup \{0\}$. Both players are discounted expected-utility maximizers, with the discount factors of $\delta_P, \delta_A \in (0, 1)$, respectively. I study perfect Bayesian equilibria of this game: players are sequentially rational and beliefs follow Bayes' rule whenever possible.

3 Why Full Discretion Is Inevitable

In Section 3.1, I show that the dynamic game has only one equilibrium outcome, which involves the Principal fully delegating project choices to the Agent and their reaching an agreement immediately. I compare this outcome to commitment benchmarks in Section 3.2.

3.1 Main Result

In this section, I show that equilibrium forces impel the Principal to effectively give the Agent full discretion and the Agent to make no compromises. To formalize this idea, I define the following:

Definition 1. *The Principal uses a* ***full discretion*** *strategy (henceforth FD) if she proposes permission set N at every history.*

I am also interested in the Agent choosing *not* to compromise at all. I define the following:

Definition 2. *The Agent uses a* ***no compromise*** *strategy (henceforth NC) if at every history, the agent of type S chooses* $\max S$ *whenever it is in the permission set, and chooses* 0 *otherwise.*

Using these definitions, I state my main result:

Theorem 1. *If the Agent is sufficiently patient ($\delta_A > \max_{i<n} \frac{\alpha_i}{\alpha_{i+1}}$), then the unique equilibrium outcome involves full discretion and no compromise.*

Theorem 1 showcases a significant loss of commitment power: in equilibrium, the Principal is forced to delegate to the Agent.

One of the key strategic forces driving Theorem 1 is that when the Agent is sufficiently patient, if the Principal does not capitulate at the outset, she cannot capitulate too fast. Otherwise, the Agent will not compromise at all.

Lemma 1. *Let the Agent be sufficiently patient ($\delta_A > \max_{i<n} \frac{\alpha_i}{\alpha_{i+1}}$). Under any equilibrium, for each Agent type S,*

> *(i) if* $\max S$ *is permitted in the current period, type S will choose project* $\max S$ *with probability one;*
>
> *(ii) if* $\max S$ *is not permitted in the current period but permitted in the next period for sure, type S with choose* 0 *with probability one.*

Proof. Part (i) is straightforward because type S can do no better than project $\max S$.

For Part (ii), since the Agent is sufficiently patient ($\delta_A > \max_{i<n} \frac{\alpha_i}{\alpha_{i+1}}$), waiting one period to obtain project $\max S$ is strictly better than accepting any other feasible project in the current period. □

Hence, to induce the Agent to compromise, the Principal cannot capitulate too soon. But then, the game will keep proceeding to the next period with positive probability.

Lemma 2. *Let the Agent be sufficiently patient ($\delta_A > \max_{i<n} \frac{\alpha_i}{\alpha_{i+1}}$). Under any equilibrium, whenever the game does not end in the first period, the game will continue to the next period with positive probability in each subsequent period at every equilibrium path history.*

Proof. Suppose to the contrary that there exists an equilibrium under which the game does not end in the first period with positive probability but ends with probability one at some equilibrium path history $\{A_0, \ldots, A_t\}$. Since this history is an equilibrium path history, there exists some Agent type S responding to all past permission sets $A_0, \ldots, A_t$ by choosing 0 with positive probability. According to Part (i) of Lemma 1, $\max S \notin \cup_{k=0}^{k=t} A_k$, which implies $\cup_{k=0}^{k=t} A_k \neq N$.

For the game to end for sure at this history, the Principal must capitulate. In other words, the Principal must permit all projects in set $N \setminus (\cup_{k=0}^{k=t} A_k)$ with probability one at this history. Suppose in an alternative case, some project $p_o \in N \setminus (\cup_{k=0}^{k=t} A_k)$ is not permitted with positive probability at this history. In other words, the Principal proposes some permission set A_{t+1} with positive probability at this history and $p_o \notin A_{t+1}$. Singleton type $\{p_o\}$ must be in the support of the Principal's belief at this history because singleton type $\{p_o\}$ has been assigned positive probability by the prior and it will choose 0 with probability one when faced with all past permission sets $A_0, \ldots, A_t$. When the Principal proposes A_{t+1} at this history, singleton type $\{p_o\}$ will also choose 0 with probability one. Therefore, the game continues to the next period with positive probability, contradicting the assumption that at this history that the game will end with probability one.

Since the Principal capitulates at history $\{A_0, \ldots, A_t\}$, the Agent will not compromise at all when the Principal proposes permission set A_t in the previous period. In other words, each type S in support of the Principal's belief at history $\{A_0, \ldots, A_{t-1}\}$ will choose project $\max S$ with probability one if it is in permission set A_t; otherwise, each type will choose 0 with probability one.[4] If $\max S \in A_t$, by Part (i) of Lemma 1, type S will choose project $\max S$ with probability one. Suppose $\max S \notin A_t$. Since type S is in support of the Principal's belief at history $\{A_0, \ldots, A_{t-1}\}$, type S must have chosen 0 with positive probability facing all past permission sets $A_0, \ldots, A_{t-1}$. Hence, by Part (i) of Lemma 1, $\max S \notin \cup_{k=0}^{k=t-1} A_k$. Thus, I have $\max S \notin \cup_{k=0}^{k=t} A_k$, which implies project $\max S$ will be permitted with probability one in the next period. Hence,

[4] Since $t-1$ appears, I need $t > 0$ to avoid a negative $t-1$. To accommodate the case $t = 0$, in this proof, let $\{A_0, \ldots, A_{-1}\}$ denote the period 0 history and let $\cup_{k=0}^{k=-1} A_k$ denote the empty set.

by Part (ii) of Lemma 1, type S will choose 0 with probability one and project $\max S$ will be selected in the next period.

Therefore, by proposing permission set A_t at history $\{A_0, \ldots, A_{t-1}\}$, the Principal receives project $\max S$ from each type S in support of her belief. Furthermore, there will be a one-period delay when the Agent is of type S with $\max S \notin A_t$. This one-period delay necessarily happens with positive probability since each singleton type $\{p_o\}$ with $p_o \notin \cup_{k=0}^{k=t} A_k$ will be in support of the Principal's belief at history $\{A_0, \ldots, A_{t-1}\}$ and such singleton type $\{p_o\}$ also satisfies $p_o \notin A_t$. The Principal will be strictly better off by permitting all projects at history $\{A_0, \ldots, A_{t-1}\}$: she continues to receive the same project from each Agent type in support of her belief but avoids this one-period delay. Hence, it is strictly suboptimal for the Principal to propose A_t at history $\{A_0, \ldots, A_{t-1}\}$, which would violate the sequential rationality of the Principal, which completes the proof. □

Now we are ready to prove Theorem 1. The key idea is as follows. Suppose there exists an equilibrium with an equilibrium outcome different from the one in Theorem 1. Under this equilibrium, the Principal cannot always fully delegate to the Agent in the first period. Hence, with positive probability, the game does not end in the first period. Then by Lemma 2, in each subsequent period, the game will keep proceeding to the next period with positive probability. Thus, it is possible that an agreement will not be reached even after a sufficiently long delay. During such a long delay, the Principal eventually realizes that she is unlikely to reach an agreement with the Agent because either there is no feasible project in her permission sets or the Agent is uncompromising; hence, she capitulates. Therefore, this equilibrium cannot exist since the Principal will eventually deviate.

Proof. It is straightforward to verify that (FD, NC) (coupled with the Principal's belief derived from Bayes' rule from the prior and the Agent's strategy) constitutes an equilibrium: since the Agent does not compromise at all under strategy NC, the best the Principal can do is to propose N. When the Principal chooses the FD strategy, by Lemma 1, the Agent will necessarily behave according to the NC strategy.

To show the uniqueness of the equilibrium outcome, suppose to the contrary that there exists an equilibrium with an equilibrium outcome different from that of equilibrium (FD, NC). I first show that under this equilibrium, there exists an equilibrium path with an arbitrarily long delay in agreement. Under this equilibrium, if the Principal permits all projects in the first period, she will reach an agreement with the Agent immediately (by Part (i) of Lemma 1) and the equilibrium outcome will be the same as under equilibrium (FD, NC). Hence, for the equilibrium outcome to be different, it must be that some project p_o is not permitted in the first period with positive probability. In other words, the Principal proposes some permission set A_0 with positive probability in period 0 and $p_o \notin A_0$. After permission set A_0 is proposed, the game will continue to the next period with positive probability since at least singlet type $\{p_o\}$ will respond to A_0 by choosing 0 with probability one. Thus, the period 1 history—$\{A_0\}$—is on equilibrium path. By Lemma 2, at history $\{A_0\}$, the game continues to the next period with positive probability. In other words, the Principal proposes some permission set A_1 with positive probability at this history and the Agent responds to A_1 by choosing 0 with positive probability. Thus, history $\{A_0, A_1\}$ is also on equilibrium path. Hence, inductively, there exists sequence $\{A_k\}_{k=0}^{k=\infty}$ such that for each t, the Principal proposes permission set A_t with positive probability at equilibrium path history $\{A_0, \ldots, A_{t-1}\}$ and the game continues to period $t+1$ with positive probability after A_t is proposed. Therefore, sequence $\{A_k\}_{k=0}^{k=\infty}$ is the equilibrium path with an arbitrarily long delay in agreement. Since it is an equilibrium path, there exists an Agent type S responding to each permission set A_t by choosing 0 with positive probability. By Part (i) of Lemma 1, $\max S \notin A_t$ for each t. Hence, $\max S \notin \cup_{k=0}^{k=\infty} A_k$, which implies $\cup_{k=0}^{k=\infty} A_k \neq N$.

Along the equilibrium path $\{A_k\}_{k=0}^{k=\infty}$, the Principal eventually realizes that the probability of reaching an agreement with the Agent is arbitrarily low. Let μ_t denote the Principal's belief at history $\{A_0, \ldots, A_{t-1}\}$; let $a_t(S)$ denote the probability of type S choosing 0 facing permission set A_t at history $\{A_0, \ldots, A_{t-1}\}$. If $S \cap A_t = \emptyset$, $a_t(S) = 1$; otherwise, type S chooses 0 with probability $a_t(S)$ and chooses project $\max S \cap A_t$ with the remaining probability. Note that it is strictly suboptimal for the Agent to choose any feasible and permitted project other than project $\max S \cap A_t$. Hence, when the Principal proposes permission set A_t at history $\{A_0, \ldots, A_{t-1}\}$, an agreement will be reached with probability

$$\sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S)).$$

This agreement probability must vanish ($\lim_{t \to \infty} \sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S)) = 0$) because:

Lemma 3. *For each Agent type S, either* $\lim_{t \to \infty} \mu_t(S) = 0$ *or* $\lim_{t \to \infty} a_t(S) = 1$.

Lemma 3 is proven in Appendix A.1. The argument is similar to the merging argument in Sorin (1999). The intuition is as follows. Consider the set of Agent types assigned positive probability by the Principal's belief (μ_t) in the limit. For each $p_o \notin \cup_{k=0}^{k=\infty} A_k$, the corresponding singleton type $\{p_o\}$ belongs to this set since it is assigned positive probability by the prior and will choose 0 with probability one in each period along equilibrium path $\{A_k\}_{k=0}^{k=\infty}$. Take any type S in this set other than singleton type $\{p_o\}$. The Principal cannot distinguish type S from singleton type $\{p_o\}$ along equilibrium path $\{A_k\}_{k=0}^{k=\infty}$. Hence, these two types must behave identically in the limit along this equilibrium path. Therefore, $\lim_{t \to \infty} a_t(S) = \lim_{t \to \infty} a_t(\{p_o\}) = 1$.

Since this agreement probability vanishes, the Principal's equilibrium payoff also vanishes. Let V_t denote the Principal's equilibrium expected discounted payoff at history $\{A_0, \ldots, A_{t-1}\}$. I show that $\lim_{t \to \infty} V_t = 0$: since the Principal proposes permission set A_t with positive probability at history $\{A_0, \ldots, A_{t-1}\}$, equilibrium payoff V_t can be written as

$$V_t = \sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S))\pi_{\max S \cap A_t} + \left[1 - \sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S))\right] \delta_P V_{t+1}.$$[5]

The bracketed term represents the probability that no agreement is reached and the game continues to the next period, which is bounded by one. Thus,

$$V_t \leq \underbrace{\sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S))}_{\text{the agreement probability}} \pi_{\max S \cap A_t} + \delta_P V_{t+1}.$$

[5] To accommodate the case $S \cap A_t = \emptyset$, let $\pi_{\max \emptyset}$ denote zero.

Since the agreement probability vanishes, for any $\varepsilon > 0$, there exists T such that

$$\sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S)) < \varepsilon \text{ for all } t > T.$$

Hence,

$$V_t \leq \varepsilon \cdot \max_p \pi_p + \delta_P V_{t+1} \text{ for all } t > T, \text{ which implies } V_t \leq \frac{\varepsilon \max_p \pi_p}{1 - \delta_P} \text{ for all } t > T.$$

Therefore, $V_t < \min_p \pi_p$ for sufficiently large t. However, by permitting all projects at history $\{A_0, \ldots, A_{t-1}\}$, the Principal obtains a payoff that is at least $\min_p \pi_p$. This contradicts the sequential rationality of the Principal and completes the proof. □

3.2 Commitment Benchmarks

Now that we understand that full discretion is inevitable in equilibria of the dynamic game, let us compare this payoff to those of relevant commitment benchmarks.

The first commitment benchmark is the static commitment setting in Armstrong and Vickers (2010), where the game always ends in one period. In this static commitment setting, the Principal can always achieve the same payoff as in equilibrium by permitting all projects. The Principal cannot do strictly better in two extreme cases. The first case is when the Principal and the Agent have the same preference order over the set of projects. The second case is when there is common knowledge that the Agent has only one feasible project—μ_0 only puts positive probability on singletons. However, as long as we are sufficiently far from these extreme cases, the Principal can do strictly better than under equilibrium. To formalize this idea, I define the following:

Definition 3. *Preferences are* ***non-congruent*** *if the worst project for the Principal does not coincide with that of the Agent:* $1 \notin \arg\min_p \pi_p$.

As long as preferences are non-congruent and the prior puts sufficiently small probability on singletons, the Principal can achieve a strictly higher payoff than under equilibrium:

Theorem 2. *The Principal can achieve a strictly higher payoff in the static commitment setting than in equilibrium if preferences are non-congruent and the prior has full support and puts sufficiently small probability on singletons.*

Theorem 2 is proven in Appendix A.2.

In this dynamic game, the more natural commitment benchmark is to allow the Principal to commit to any strategy. Clearly, any static commitment payoff is achievable by committing to the same permission set in each period. Furthermore, whenever the Principal can achieve a strictly higher payoff in the static commitment setting than in equilibrium, she can also achieve a strictly higher payoff in this dynamic commitment setting than in the static commitment setting. Let A^* denote the optimal permission set in the static commitment setting. The Principal can commit to set A^* for the first T periods and then give full discretion right afterward. As long as T is sufficiently large, the Agent will behave myopically in the first period facing permission set A^*. Whenever an agreement is not reached in the first period, the Principal retains the chance to reach an agreement with the Agent in period T. Hence, the Principal obtains a strictly higher payoff in this setting than in the static commitment setting.

In the dynamic commitment setting, the Principal's payoff is bounded from above by the **first best payoff**: the payoff the Principal will receive if she perfectly observes the Agent's type,

$$\overline{V} \equiv \sum_{S} \mu_0(S) \max_{p \in S} \pi_p.$$

If the Principal is arbitrarily patient, she can achieve a payoff arbitrarily close to the first best payoff:

Theorem 3. *By committing to a strategy of the dynamic game, the Principal can achieve a payoff arbitrarily close to the first best payoff if she is sufficiently patient. In other words, for any $\varepsilon > 0$ and $\delta_A \in (0, 1)$, there exists $\underline{\delta} \in (0, 1)$ such that the Principal can achieve a payoff above $\overline{V} - \varepsilon$ if $\delta_P > \underline{\delta}$.*

Theorem 3 is proven in Appendix A.3.

4 Related Literature

My model builds on project selection models studied by Armstrong and Vickers (2010) and Nocke and Whinston (2013). In their models, the Principal interacts with the Agent in a one-shot manner, which automatically grants the Principal commitment power. In my model, however, the Principal cannot commit in advance to future permission sets. I find that when the Principal cannot commit, she is forced to fully delegate project choice to the Agent under all generic prior distributions and regardless of how misaligned their preferences are. In their models, the Principal does not give the Agent full discretion except in non-generic cases.

There are other papers in the literature providing rationales for delegation. Aghion and Tirole (1997) show that superiors in organizations will delegate authority to subordinates to encourage initiatives from subordinates. Dessein (2002) shows a Principal will prefer delegating control to an informed Agent rather than communicating with that Agent as long as their incentives are not too misaligned. I identify a new and distinct force driving delegation of control: in my setting, the Principal delegates project choice to the Agent when she is uncertain what the Agent can do and she cannot commit in advance to her future restrictions. This force drives full delegation to be the unique equilibrium outcome regardless of how misaligned the Agent's and Principal's incentives are.

My result relates to the classic Coase conjecture (Coase, 1972) in the bargaining literature. In the Coasian bargaining setting, a monopoly seller with a durable good bargains with a buyer and does not know the value the buyer places on the seller's good. The monopoly seller proposes the price in each period. Gul, Sonnenschein, and Wilson (1986) show that in the gap case in which the lowest possible value of the buyer is strictly positive, the unique equilibrium is stationary and satisfies the Coase conjecture as the period length converges to zero: (i) all gains from trade are realized almost instantaneously; and (ii) the seller's profit converges to the the lowest possible value the buyer assigns to the good. They show the same is true in the no-gap case (in which the lowest possible value of the buyer equals zero) if only stationary equilibria are considered.[6]

[6] Ausubel and Deneckere (1989) show that in the no-gap case, there also exists a non-

Though my result appears to be similar to that of the Coase conjecture, there are notable differences. First, in equilibrium, the Coasian bargaining game takes a finite number of periods to end while my game ends exactly at the end of the first period. Second, my setting is fully discrete and all equilibria involve full discretion; non-Coasian equilibria emerge in the Coasian bargaining setting when the price space is discrete. Third, in the Coasian bargaining setting, randomization (of the buyer) is required to support the equilibrium.[7] In contrast, the equilibrium in my setting consists of pure strategies; no randomization is involved.

My problem is categorized as a problem with limited commitment. In the mechanism design literature, Doval and Skreta (2022) study dynamic mechanism-selection games and assume that the designer can commit only to short-term mechanisms. They develop a revelation principle in this setting. To the best of my knowledge, there are no other papers in the delegation literature that study delegation with limited commitment. The only exception is the recent working paper by Mallick and Teoman (2022). In their model, in each period, the Agent moves first to propose a project to the Principal and the Principal then either accepts or rejects the Agent's proposal. They find that under some conditions, there exists an equilibrium under which the Principal attains the commitment payoff.

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stationary "reputation equilibrium" in which the seller obtains his commitment payoff.

[7] In the Coasian bargaining game, to support equilibrium, the buyer needs to randomize at least off equilibrium path. See Gul, Sonnenschein, and Wilson (1986). When value distribution is discrete, randomization on equilibrium path is also needed.

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A Appendix

A.1 Proof of Lemma 3

Proof. Recall that μ_t is the Principal's belief at history $\{A_0, \dots, A_{t-1}\}$ and $a_t(S)$ is the probability type S choosing 0 when the Principal proposes permission set A_t at history $\{A_0, \dots, A_{t-1}\}$. Hence, both μ_t and a_t are deterministic functions.

Take any type S such that $\lim_{t\to\infty} a_t(S) = 1$ is not true. There exists a subsequence $\{a_{t_k}(S)\}$ converging to a limit other than one—$\lim_{k\to\infty} a_{t_k}(S) = a_\infty < 1$. Hence, I can find T that

$$a_{t_k}(S) \leq \frac{1 + a_\infty}{2} \text{ for all } t_k \geq T. \tag{1}$$

Let Q_t denote the probability that the Agent chooses 0 facing all permission sets $A_0, \dots, A_{t-1}$ in past periods. By Bayes' rule:

$$\mu_t(S) = \frac{\mu_0(S)\Pi_{j=0}^{j=t-1} a_j(S)}{Q_t}.$$

Since $\cup_{k=0}^{k=\infty} A_k \neq N$, there exists singleton type $\{p_o\}$ with $p_o \notin \cup_{k=0}^{k=\infty} A_k$. By Bayes' rule:

$$\mu_t(\{p_o\}) = \frac{\mu_0(\{p_o\})\Pi_{j=0}^{j=t-1} a_j(\{p_o\})}{Q_t} = \frac{\mu_0(\{p_o\})}{Q_t}.$$

The second equality is because for all j, $a_j(\{p_o\}) = 1$ since $p_o \notin A_j$.

Consider the following likelihood ratio:

$$\frac{\mu_t(S)}{\mu_t(\{p_o\})} = \frac{\mu_0(S)\Pi_{j=0}^{j=t-1} a_j(S)}{\mu_0(\{p_o\})} \leq \frac{\mu_0(S)}{\mu_0(\{p_o\})} \Big[\frac{1 + a_\infty}{2}\Big]^{|\{t_k | T \leq t_k < t\}|},$$

where the inequality is by Equation (1) and $a_j(S) \leq 1$.

Notice that $\dfrac{1 + a_\infty}{2} < 1$. Hence, $\lim_{t\to\infty} \dfrac{\mu_t(S)}{\mu_t(\{p_o\})} = 0$, which implies $\lim_{t\to\infty} \mu_t(S) = 0$. This completes the proof. $\square$

A.2 Proof of Theorem 2

Proof. In this static commitment setting, if the Principal permits all projects, she will receive the same payoff as in equilibrium. If the Principal permits all but her least preferred project, she will obtain a strictly higher payoff: Denote her least preferred project by $\underline{p}$. When the Principal permits all projects, each type S is going to choose project $\max S$. By removing project $\underline{p}$ from the permission set, the Principal is going to obtain the same project if $\underline{p}$ is not the Agent's favorite feasible project; the Principal is going to make a loss of $\pi_{\underline{p}}$ if the Agent is of singleton type $\{\underline{p}\}$; the Principal's payoff will increase by $\pi_{\max S \setminus \{\underline{p}\}} - \pi_{\underline{p}}$ if the Agent is of type S other than singleton type $\{\underline{p}\}$ and that $\underline{p}$ is exactly the Agent's favorite feasible project ($\max S = \underline{p}$). Hence, removing project $\underline{p}$ will strictly increase the Principal's payoff if the total loss is smaller than the total increase in her payoff:

$$\mu_0(\{\underline{p}\})\pi_{\underline{p}} < \sum_{S: S \neq \{\underline{p}\}, \max S = \underline{p}} \mu_0(S)(\pi_{\max(S \setminus \{\underline{p}\})} - \pi_{\underline{p}}).$$

Since preferences are non-congruent, $\underline{p} \neq 1$. Agent type $\{1, \underline{p}\}$ will be in set $\{S : S \neq \{\underline{p}\}, \max S = \underline{p}\}$, and $\pi_1 - \pi_{\underline{p}} > 0$ since $\underline{p}$ is the Principal's least preferred project and preferences are non-congruent. In addition, $\mu_0(\{1, \underline{p}\}) > 0$ since the prior has full support. Hence, the right-hand side of the above inequality is strictly positive. As long as the prior put sufficiently less probability on singleton $\{\underline{p}\}$, the above inequality will hold, and the Principal can achieve a strictly higher payoff in the static commitment setting than in equilibrium. $\square$

A.3 Proof of Theorem 3

Proof. Let T be a sufficiently large number such that no Agent type is willing to wait T periods to obtain a better project.

$$T \equiv \min\{t | \alpha_n \delta_A^t < \alpha_1\}.$$

The Principal commits to the following **Slowly Compromising Strategy**: Order the set of the Principal's payoff from potential projects ($\{\pi_p | p = 1, \ldots, n\}$)

in a decreasing manner $\pi_{(1)} > \cdots > \pi_{(k)} > \cdots > \pi_{(K)}$. In the first T periods, only permits projects with a payoff to the Principal at least $\pi_{(1)}$. For each $k = 1, \ldots, K-1$, in the k-th T periods (from period $(k-1)T$ to period $kT-1$), permits projects with payoffs to the Principal at least $\pi_{(k)}$. From period $(K-1)T$ on, permits all projects. For each Agent type S, if the Principal's favorite project in set S gives her a payoff $\pi_{(k)}$ ($\max_{p \in S} \pi_p = \pi_{(k)}$), then under the Slowly Compromising Strategy, type S has no feasible project permitted before period $(k-1)T$. In period $(k-1)T$, all permitted projects in set S give the Principal payoff $\pi_{(k)}$; and by definition of T, the Agent has no incentive to wait for a better project. Hence, the Principal will receive payoff $\delta_P^{(k-1)T}\pi_{(k)}$ if the Agent has type S. Since T is independent of δ_P, this payoff will converge to $\pi_{(k)} = \max_{p \in S} \pi_p$ as δ_P goes to one . As a result, the Principal's payoff under the Slowly Compromising Strategy converges to the First Best Payoff as δ_P goes to one. $\square$