

Full Discretion is Inevitable*

Wenhao Li[†]

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Abstract

This article studies a dynamic project-selection game between a Principal and an Agent with conflicting interests. Only the Agent knows what projects are feasible. In each period before a project is selected, the Principal imposes a restriction set. The Agent can select any feasible project within this set, thereby ending the game. The Agent can also stay silent, in which case the game will proceed to the next period. Importantly, the Principal cannot commit to her future restriction sets. I show that when the Agent is sufficiently patient, the Principal fully delegates to the Agent in the unique equilibrium.

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[†]European University Institute (e-mail: wenhao.econ@gmail.com).

1 Introduction

Consider the following incentive problem: a Principal (she) and an Agent (he) would like to agree on a project but only the Agent knows which projects are available. This kind of “project-choice” problem is ubiquitous: it manifests in the relationship between a university and a department when determining whom to hire or between a firm’s division and its headquarters when considering a potential investment. The Agent in these examples is better informed about the availability of each project than the Principal, but their preferences may not align. The Principal then has to decide to what degree she should delegate the choice to the Agent. Full delegation always results in the selection of a project whenever one is feasible but results in the Agent also choosing his favorite project. By contrast, restricting the Agent to some subset of projects may tilt the Agent towards projects preferred by the Principal but also increases the chance that no project is selected, resulting in the status quo outcome.

In their seminal work, [Armstrong and Vickers \(2010\)](#) study this project-choice problem as a one-shot interaction: a Principal commits to a “restriction” wherein the Agent can propose a project only in a given set. Such a proposal is then selected. If the Agent has no project to propose in that restriction, both parties obtain the status quo. An important idea that emerges from this analysis is that to counteract the Agent’s bias, the Principal may exclude projects that reflect Pareto gains relative to the status quo. Partial delegation thus emerges as an optimal solution.

This article revisits this setting but with dynamic considerations in mind. Suppose that the Agent does not recommend a project in the Principal’s re-

striction. The Principal may then infer that the projects in her restriction are infeasible. She may wish to give the Agent another chance by adjusting the restriction. Anticipating this response, the Agent may then hold back on proposing projects. How well can the Principal do in this setting?

The main result of this article answers this question. Informally stated, the answer is as follows:

Theorem. *If the Agent is patient, the unique equilibrium outcome involves the Principal fully delegating to the Agent.*

The key idea is that if the Principal does not fully delegate to the Agent from the start, the Agent has an incentive to hold back on proposing projects in the restricted set so as to convince the Principal that such projects are infeasible. At some point, the Principal capitulates, letting the Agent choose. If the Agent is sufficiently patient, he will wait until that happens. The Principal is then trapped and would have been better off by capitulating at the outset.

This strategic logic is reminiscent of the reputational arguments that feature in [Kreps and Wilson \(1982\)](#) and [Milgrom and Roberts \(1982\)](#) or the Coase Conjecture ([Gul, Sonnenschein, and Wilson, 1986](#)). I highlight some important distinctions. Relative to reputation models, all players in my setting are strategic; more critically, I use a distinct argument to show the uniqueness of equilibrium as my model has an infinite horizon. Relative to the Coase Conjecture, types in my model are not ordered, and hence there is no analog to single crossing or the skimming property being satisfied here.¹ Second, the actions in this model

¹Indeed, in a different setting, [Ali, Kartik, and Kleiner \(2023\)](#) show that failures of standard single crossing may lead to equilibria that fail the skimming property and result in the commitment payoff being attained.

are discrete—i.e., to approve or reject a project—yet the aforementioned outcome emerges across all equilibria. By contrast, in seller-buyer bargaining with discrete prices, non-Coasian equilibria are possible.²

Existing articles rationalize delegation by highlighting the benefits to the Principal. [Aghion and Tirole \(1997\)](#) show delegation encourages the Agent to take initiative, whereas [Dessein \(2002\)](#) show delegating to an informed Agent is better for the Principal than communicating with the Agent. This article complements the existing work by identifying a new and distinct force: the Principal is forced to fully delegate project choice to the Agent because she is uncertain about what the Agent can do and she cannot commit in advance to her future restrictions.

The rest of the article is organized as follows. [Section 2](#) introduces the setup. [Section 3](#) presents the main result of this article (i.e., full discretion is inevitable in equilibrium) and compares the equilibrium outcome to relevant commitment benchmarks. [Section 4](#) discusses related literature. All proofs are in the appendices.

2 Model

A Principal (she) and an Agent (he) jointly choose a project. The set of potential projects is denoted by $N \equiv \{1, \dots, n\}$, and the set of all non-empty subsets of N is denoted by \mathcal{N} . If project p is selected, then the Principal's payoff is π_p and the Agent's payoff is α_p ; if no project is selected, then each player obtains 0. I

²[Von der Fehr and Kühn \(1995\)](#) discusses that, when the buyer is sufficiently patient, there exists a stationary equilibrium for each price p smaller than the buyer's lowest value, wherein the good is sold immediately at price p . Additionally, there are trigger-strategy equilibria.

consider the generic case in which the Agent has strict preference over the set of projects, and I order projects so that α_p is strictly increasing in p . I further assume that for each p , π_p and α_p are in $(0, 1)$. In my setting these are without loss of generality, as the Principal would never permit projects in which she obtains a negative payoff and the Agent would never choose projects where he obtains a negative payoff.

The Principal and the Agent both know π_p and α_p for each potential project p .³ The informational friction is that the Principal does not know which projects are feasible for the Agent; this is private information possessed by the Agent. The set of feasible projects, $S \subseteq N$, is drawn according to the prior, $\mu_0 \in \Delta(\mathcal{N})$. Because \mathcal{N} excludes the empty set, this presumes that there is always at least one project that is feasible. For expositional convenience, I further assume that μ_0 has positive probability on any singleton: for every p , $\mu_0(\{p\}) > 0$. Note that this assumption is milder than a full-support assumption.⁴ I refer to S as the Agent's *type*. Given a set of feasible projects, S , $\max S$ (resp. $\min S$) denotes the feasible project that has the highest (resp. lowest) label.

I model a dynamic delegation game. The Agent's type, S , is drawn from the prior distribution, μ_0 , before the start of the game and remains fixed throughout the game. At every period $t = 0, 1, \dots$, until a project is selected:

1. The Principal proposes permission set $A_t \subseteq N$.

³I will later discuss that this assumption is without loss. The main result continues to hold if, as in [Armstrong and Vickers \(2010\)](#), the Principal only knows the payoff of the project selected by the Agent.

⁴In the Supplementary Appendix, I demonstrate that without this assumption, numerous other equilibrium outcomes exist. However, the main result remains robust because these alternative equilibrium outcomes are fragile, even to slight uncertainty about project feasibility.

2. The Agent chooses from set $(A_t \cap S) \cup \{0\}$.⁵ Here, action 0 represents that the Agent stays silent.

- If the Agent chooses project p in $A_t \cap S$, the game ends with project p selected. I refer to this case as reaching an **agreement** on project p .
- If the Agent chooses 0, the game continues to period $t + 1$.

In this game a history is a sequence of permission sets. A strategy for the Principal is a function that assigns to every history a probability distribution over 2^N , interpreted as the (possibly random) permission set the Principal proposes given that the Agent has chosen 0 in all past periods. A strategy for the Agent is a function that—for each history, each type S , and each current permission set A —specifies the probability the Agent chooses each action in set $(A \cap S) \cup \{0\}$. Both players are discounted expected-utility maximizers, with discount factors of $\delta_P, \delta_A \in (0, 1)$, respectively. I study perfect Bayesian equilibria of this game: players are sequentially rational and beliefs follow Bayes’ rule whenever possible.

3 Why Full Discretion Is Inevitable

In the first subsection, I present my main result: the dynamic game has only one equilibrium outcome, which involves the Principal fully delegating project choices to the Agent and them reaching an agreement immediately. I compare this outcome to commitment benchmarks in the second subsection.

⁵I implicitly assume the Agent cannot choose non-feasible or non-permitted projects. The former is without loss in contexts where non-feasible projects incur losses to the Agent. Whether my main result remains robust when allowing for the latter is an open question.

Main Result

In this section, I show that equilibrium forces impel the Principal to effectively give the Agent full discretion and the Agent to make no compromises. To formalize this idea, I define the following:

Definition 1. *The Principal uses a **full discretion** strategy (henceforth FD) if she proposes permission set N at every history. The Agent uses a **no compromise** strategy (henceforth NC) if at every history, the agent of type S chooses $\max S$ whenever it is in the permission set, and chooses 0 otherwise.*

Using these definitions, I state my main result. Let $\bar{\delta} \equiv \max_{p < n} \frac{\alpha_p}{\alpha_{p+1}}$.

Theorem 1. *If the Agent is sufficiently patient—, $\delta_A > \bar{\delta}$, then the unique equilibrium outcome involves full discretion and no compromise. In other words, immediate full discretion is inevitable.*

The key intuition is as follows: If the Principal does not capitulate at the outset, she cannot capitulate in the second period. Otherwise, the Agent, being patient, will not compromise at all, expecting to obtain his preferred project shortly. Similarly, if the Principal does not capitulate in period t , she cannot capitulate in period $t + 1$. Thus, under a hypothetical equilibrium in which the Principal does not capitulate at the outset, the game will continue to the next period with positive probability. Consequently, it is possible that an agreement is not reached even after a sufficiently long delay. During such a long delay, after observing the Agent choosing 0 repeatedly, the Principal eventually realizes that either there is no feasible project in her permission sets or the Agent is uncompromising. Hence, she is unlikely to reach an agreement with the Agent

under her equilibrium permission sets. The Principal is better off if she deviates to permit all projects. Therefore, if the Agent is sufficiently patient, the Principal must immediately capitulate.

I conclude this subsection by providing three comments about [Theorem 1](#). First, note that [Theorem 1](#) only requires the Agent to be patient. This does not imply that the theorem relies on the Agent's discount factor approaching one while keeping the Principal's discount factor fixed. In fact, the theorem remains valid even if both the Principal and the Agent have the same discount factor ($\delta_P = \delta_A = \delta > \bar{\delta}$). It also holds true when both the Principal and the Agent are very patient but have different discount factors.⁶

Secondly, [Theorem 1](#) does not rely on the Principal knowing the characteristics or payoffs of all potential projects. [Theorem 1](#) remains valid if, as in [Armstrong and Vickers \(2010\)](#), the Principal only knows the payoff vector of the project selected by the Agent. To accommodate this possibility, let $C \equiv \{(\pi_1, \alpha_1), \dots, (\pi_n, \alpha_n)\}$ denote the set of possible payoff vectors. Now suppose the payoff vector of each potential project is specified by a mapping between N and C , and any bijection between N and C can be this mapping with positive probability.⁷ The Agent privately observes the realized payoff mapping. The Principal does not know the realized payoff mapping, hence she can only specify a permission set as a subset of C . The project chosen by the Agent must have a payoff vector within the Principal's permission set, reflecting the idea that the Principal can verify the characteristics of the project selected by the Agent. In

⁶For example, let's assume that each period lasts for a duration of Δ , and the Principal and the Agent have discount rates of r_P and r_A , respectively. Therefore, $\delta_P = e^{-r_P \Delta}$ and $\delta_A = e^{-r_A \Delta}$. If Δ is sufficiently small and $\delta_A > \bar{\delta}$, [Theorem 1](#) remains valid.

⁷Note that in this new setting, the payoff vector of project p is not necessarily (π_p, α_p) ; it can be any element in C .

this new setting, the same analysis holds and [Theorem 1](#) continues to be valid.

Lastly, [Theorem 1](#) remains valid even if the Principal commits to giving the Agent only one chance with any project. Formally, in period t , at any history $\{A_0, \dots, A_{t-1}\}$, the permission set A_t proposed by the Principal cannot intersect any previous permission set. In this new setting, [Theorem 1](#) remains valid, and the same analysis applies. Intuitively, if the Agent's preferred project is not in the current permission set, it can still be included in the next period's permission set. Hence, the Agent still has incentives to hold back on proposing projects, and the Principal faces the same problem as before.

Commitment Benchmarks

Now that we understand that full discretion is inevitable in equilibria of the dynamic game, let us compare this payoff to those of relevant commitment benchmarks.

The first commitment benchmark is the one-shot game studied by [Armstrong and Vickers \(2010\)](#). In this one-shot game, the Principal commits to a permission set within which the Agent can propose a feasible project. Such a proposal is then selected. If the Agent has no feasible project to propose in that set, both parties obtain the status quo. I refer to this setting as the static commitment setting. In this static commitment setting, the Principal can always achieve the same payoff as in equilibrium by permitting all projects. The Principal cannot do strictly better in two extreme cases. The first case is when the Principal and the Agent have the same preference order over the set of projects. The second case is when there is common knowledge that the Agent has only one feasible

project— μ_0 only puts positive probability on singletons. However, as long as we are sufficiently far from these extreme cases, the Principal can do strictly better than under equilibrium. To formalize this idea, I define the following:

Definition 2. *Preferences are **non-congruent** if the worst project for the Principal does not coincide with that of the Agent: $1 \notin \arg \min_p \pi_p$.*

As long as preferences are non-congruent and the prior puts sufficiently small probability on singletons, the Principal can achieve a strictly higher payoff than under equilibrium:

Theorem 2. *The Principal can achieve a strictly higher payoff in the static commitment setting than in equilibrium if preferences are non-congruent and the prior has full support and puts sufficiently small probability on singletons.*

Relative to the static commitment setting, a more natural commitment benchmark in this dynamic game is to allow the Principal to commit to any strategy. Clearly, any static commitment payoff is achievable by committing to the same permission set in each period. If the Principal is arbitrarily patient, she can achieve a payoff arbitrarily close to the **first best payoff**: the payoff the Principal will receive if she perfectly observes the Agent's type,

$$\bar{V} \equiv \sum_S \mu_0(S) \max_{p \in S} \pi_p.$$

Theorem 3. *By committing to a strategy of the dynamic game, the Principal can achieve a payoff arbitrarily close to the first best payoff if she is sufficiently patient. In other words, for any $\varepsilon > 0$ and $\delta_A \in (0, 1)$, there exists $\underline{\delta} \in (0, 1)$ such that the Principal can achieve a payoff above $\bar{V} - \varepsilon$ if $\delta_P > \underline{\delta}$.*

4 Related Literature

My model builds on project-selection models studied by [Armstrong and Vickers \(2010\)](#) and [Nocke and Whinston \(2013\)](#). In their models, the Principal interacts with the Agent in a one-shot manner, which automatically grants the Principal commitment power. In my model, however, the Principal cannot commit in advance to future permission sets. I find that when the Principal cannot commit, she is forced to fully delegate project choice to the Agent under all generic prior distributions and regardless of how misaligned their preferences are. In their models, the Principal does not give the Agent full discretion except in non-generic cases.

My work is far from the first to rationalize delegation. [Aghion and Tirole \(1997\)](#) show that superiors in organizations will delegate authority to subordinates to encourage initiatives from subordinates. [Dessein \(2002\)](#) shows a Principal will prefer delegating control to an informed Agent rather than communicating with that Agent as long as their incentives are not too misaligned. I identify a new and distinct force that pushes towards delegation: the Principal delegates project choice to the Agent when she is uncertain what the Agent can do and cannot commit in advance to her future restrictions. This force drives full delegation to be the unique equilibrium outcome regardless of how misaligned the Agent's and Principal's incentives are.

My result relates to the classic Coase conjecture [Coase \(1972\)](#) in the bargaining literature and to the reputational arguments featured in [Kreps and Wilson \(1982\)](#) and [Milgrom and Roberts \(1982\)](#). Essentially, my result can be viewed as a Coase conjecture or a reputational result in the delegated project choice set-

ting. Interestingly, even though my model does not have any behavioral types, the project-selection framework allows the Agent to build a reputation for being uncompromising.

My problem is categorized as a problem with limited commitment. In the mechanism design literature, [Doval and Skreta \(2022\)](#) study dynamic mechanism-selection games and assume that the designer can commit only to short-term mechanisms. They develop a revelation principle in this setting. To the best of my knowledge, there are no other articles in the delegation literature that study delegation with limited commitment. The only exception is the recent article by [Mallick and Teoman \(2022\)](#). In their model, in each period, the Agent moves first to propose a project to the Principal and the Principal then either accepts or rejects the Agent’s proposal. They find that under some conditions, there exists an equilibrium under which the Principal attains the commitment payoff.

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Appendix

To prove [Theorem 1](#), I first observe that if the Principal capitulates too quickly, the Agent will not compromise at all. This observation is formalized in [Lemma 1](#):

Lemma 1. *Let the Agent be sufficiently patient ($\delta_A > \bar{\delta}$). Under any equilibrium, for each Agent type S ,*

(i) if $\max S$ is permitted in the current period, type S will choose project $\max S$ with probability one;

(ii) if $\max S$ is not permitted in the current period but permitted in the next period for sure, type S will choose 0 with probability one.

The lemmas used in the proof of [Theorem 1](#) will be proved following this proof. [Lemma 1](#) has an important implication: If the Principal does not end the game in the first period by fully delegating to the Agent, the game will keep proceeding to the next period with positive probability. This is established in [Lemma 2](#):

Lemma 2. *Let the Agent be sufficiently patient ($\delta_A > \bar{\delta}$). Under any equilibrium, whenever the game does not end in the first period, the game will continue to the next period with positive probability in each subsequent period at every equilibrium path history.*

Now we are ready to prove [Theorem 1](#). It is straightforward to verify that (FD, NC) (coupled with the Principal's belief derived from Bayes' rule from the prior and the Agent's strategy) constitutes an equilibrium: because the Agent does not compromise at all under strategy NC, the best the Principal can do is

to propose N . When the Principal chooses the FD strategy, by Lemma 1, the Agent will necessarily behave according to the NC strategy.

To show the uniqueness of the equilibrium outcome, suppose to the contrary that there exists an equilibrium with an equilibrium outcome different from that of equilibrium (FD, NC). I first show that under this equilibrium, there exists an equilibrium path with an arbitrarily long delay in agreement. Under this equilibrium, if the Principal permits all projects in the first period, she will reach an agreement with the Agent immediately (by Part (i) of Lemma 1) and the equilibrium outcome will be the same as under equilibrium (FD, NC). Hence, for the equilibrium outcome to be different, it must be that some project \tilde{p}_o is not permitted in the first period with positive probability. In other words, the Principal proposes some permission set \tilde{A}_0 with positive probability in period 0 and $\tilde{p}_o \notin \tilde{A}_0$. After permission set \tilde{A}_0 is proposed, the game will continue to the next period with positive probability because at least singleton type $\{\tilde{p}_o\}$ will respond to \tilde{A}_0 by choosing 0 with probability one. Thus, the period 1 history— $\{\tilde{A}_0\}$ —is on equilibrium path. By Lemma 2, at history $\{\tilde{A}_0\}$, the game continues to the next period with positive probability. In other words, the Principal proposes some permission set \tilde{A}_1 with positive probability at this history and the Agent responds to \tilde{A}_1 by choosing 0 with positive probability. Thus, history $\{\tilde{A}_0, \tilde{A}_1\}$ is also on equilibrium path. Hence, inductively, there exists sequence $\{\tilde{A}_k\}_{k=0}^{k=\infty}$ such that for each t , the Principal proposes permission set \tilde{A}_t with positive probability at equilibrium path history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$ and the game continues to period $t + 1$ with positive probability after \tilde{A}_t is proposed. In other words, sequence $\{\tilde{A}_k\}_{k=0}^{k=\infty}$ is the equilibrium path with an arbitrarily long delay in agreement. Note that for this to happen, there must exist an Agent type \tilde{S}

who responds to each permission set \tilde{A}_t by choosing 0 with positive probability. By Part (i) of [Lemma 1](#), $\max \tilde{S} \notin \tilde{A}_t$ for each t . Hence, $\max \tilde{S} \notin \cup_{k=0}^{k=\infty} \tilde{A}_k$, which implies $\cup_{k=0}^{k=\infty} \tilde{A}_k \neq N$.

Along the equilibrium path $\{\tilde{A}_k\}_{k=0}^{k=\infty}$, the Principal eventually realizes that the probability of reaching an agreement with the Agent is arbitrarily low. Let μ_t denote the Principal's belief at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$; let $a_t(S)$ denote the probability of type S choosing 0 when facing permission set \tilde{A}_t at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$. If $S \cap \tilde{A}_t = \emptyset$, $a_t(S) = 1$; otherwise, type S chooses 0 with probability $a_t(S)$ and chooses project $\max(S \cap \tilde{A}_t)$ with the remaining probability. Note that it is strictly suboptimal for the Agent to choose any feasible and permitted project other than project $\max(S \cap \tilde{A}_t)$. Hence, when the Principal proposes permission set \tilde{A}_t at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$, an agreement will be reached with probability

$$\sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S)).$$

This agreement probability must vanish ($\lim_{t \rightarrow \infty} \sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S)) = 0$) because:

Lemma 3. *For each Agent type S , either $\lim_{t \rightarrow \infty} \mu_t(S) = 0$ or $\lim_{t \rightarrow \infty} a_t(S) = 1$.*

Because this agreement probability vanishes, the Principal's equilibrium payoff also vanishes. Let \tilde{V}_t denote the Principal's equilibrium expected discounted payoff at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$. I show that $\lim_{t \rightarrow \infty} \tilde{V}_t = 0$: Because the Principal proposes permission set \tilde{A}_t with positive probability at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$,

equilibrium payoff \tilde{V}_t can be written as

$$\tilde{V}_t = \sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S)) \pi_{\max S \cap \tilde{A}_t} + \left[1 - \sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S)) \right] \delta_P \tilde{V}_{t+1}.^8$$

The bracketed term represents the probability that no agreement is reached and the game continues to the next period, which is bounded by one. Thus,

$$\tilde{V}_t \leq \underbrace{\sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S)) \pi_{\max S \cap \tilde{A}_t}}_{\text{the agreement probability}} + \delta_P \tilde{V}_{t+1}.$$

Because the agreement probability vanishes, for any $\varepsilon > 0$, there exists T such that

$$\sum_{S \in \mathcal{N}} \mu_t(S)(1 - a_t(S)) < \varepsilon \text{ for all } t > T.$$

Hence,

$$\tilde{V}_t \leq \varepsilon \cdot \max_p \pi_p + \delta_P \tilde{V}_{t+1} \text{ for all } t > T, \text{ which implies } \tilde{V}_t \leq \frac{\varepsilon \max_p \pi_p}{1 - \delta_P} \text{ for all } t > T.$$

Therefore, $\tilde{V}_t < \min_p \pi_p$ for sufficiently large t . However, by permitting all projects at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$, the Principal obtains a payoff that is at least $\min_p \pi_p$. This contradicts the sequential rationality of the Principal and completes the proof.

Proof of Lemma 1

Proof. Part (i) is straightforward because type S can do no better than project

⁸To accommodate the case $S \cap \tilde{A}_t = \emptyset$, let $\pi_{\max \emptyset}$ denote zero.

$\max S$.

For Part (ii), because the Agent is sufficiently patient ($\delta_A > \bar{\delta}$), waiting one period to obtain project $\max S$ is strictly better than accepting any other feasible project in the current period. \square

Proof of Lemma 2

Proof. Suppose to the contrary that there exists an equilibrium under which the game does not end in the first period with positive probability but ends with probability one at some equilibrium path history $\{\tilde{A}_0, \dots, \tilde{A}_t\}$. Because this history is an equilibrium path history, there exists some Agent type \tilde{S} responding to all past permission sets $\tilde{A}_0, \dots, \tilde{A}_t$ by choosing 0 with positive probability. According to Part (i) of Lemma 1, $\max \tilde{S} \notin \cup_{k=0}^{k=t} \tilde{A}_k$, which implies $\cup_{k=0}^{k=t} \tilde{A}_k \neq N$.

I claim that for the game to end for sure at this history, the Principal must capitulate. In other words, the Principal must permit all projects in the set $N \setminus (\cup_{k=0}^{k=t} \tilde{A}_k)$ with probability one at this history. Suppose, alternatively, that some project $p_o \in N \setminus (\cup_{k=0}^{k=t} \tilde{A}_k)$ is not permitted with positive probability at this history. In other words, the Principal proposes some permission set \hat{A}_{t+1} with positive probability at this history and $p_o \notin \hat{A}_{t+1}$. Singleton type $\{p_o\}$ must be in the support of the Principal's belief at this history because singleton type $\{p_o\}$ has been assigned positive probability by the prior and will choose 0 with probability one when faced with all past permission sets $\tilde{A}_0, \dots, \tilde{A}_t$. When the Principal proposes \hat{A}_{t+1} at this history, singleton type $\{p_o\}$ will also choose 0 with probability one. Therefore, the game continues to the next period with positive probability, contradicting the assumption that the game will end with probability one at this history.

I then show that the Agent will not compromise at all when the Principal proposes permission set \tilde{A}_t in the previous period. In other words, each type S in the support of the Principal's belief at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$ will choose project $\max S$ with probability one if it is in permission set \tilde{A}_t ; otherwise, each type will choose 0 with probability one.⁹ To see this, note that if $\max S \in \tilde{A}_t$, by Part (i) of [Lemma 1](#), type S will choose project $\max S$ with probability one. Suppose $\max S \notin \tilde{A}_t$. Because type S is in the support of the Principal's belief at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$, type S must have chosen 0 with positive probability when facing all past permission sets $\tilde{A}_0, \dots, \tilde{A}_{t-1}$. Hence, by Part (i) of [Lemma 1](#), $\max S \notin \cup_{k=0}^{t-1} \tilde{A}_k$. Thus, $\max S \notin \cup_{k=0}^t \tilde{A}_k$, which implies project $\max S$ will be permitted with probability one in the next period. Hence, by Part (ii) of [Lemma 1](#), type S will choose 0 with probability one and project $\max S$ will be selected in the next period.

Therefore, by proposing \tilde{A}_t at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$, for each type S in the support of the Principal's belief, the Principal receives project $\max S$ immediately if $\max S \in \tilde{A}_t$; otherwise, the Principal receives project $\max S$ after a one-period delay if $\max S \notin \tilde{A}_t$. This one-period delay necessarily happens with positive probability because each singleton type $\{p_o\}$ with $p_o \notin \cup_{k=0}^t \tilde{A}_k$ will be in the support of the Principal's belief and satisfies $\max\{p_o\} \notin \tilde{A}_t$. The Principal will be strictly better off by permitting all projects at this history because she continues to receive the same project from each Agent type in support of her belief but avoids this one-period delay. In other words, proposing \tilde{A}_t at this history is strictly suboptimal, violating the sequential rationality of the Principal.

⁹Because $t - 1$ appears, I need $t > 0$ to avoid a negative $t - 1$. To accommodate the case $t = 0$, in this proof, let $\{\tilde{A}_0, \dots, \tilde{A}_{-1}\}$ denote the period 0 history and let $\cup_{k=0}^{t-1} \tilde{A}_k$ denote the empty set.

This completes the proof. □

Proof of Lemma 3 The argument is similar to the merging argument in Sorin (1999). Before presenting the formal proof, let me provide some intuition. I showed that $\cup_{k=0}^{\infty} \tilde{A}_k \neq N$. Hence, there exists a singleton type $\{p_o\}$ with $p_o \notin \cup_{k=0}^{\infty} \tilde{A}_k$. In each period t at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$, the singleton type $\{p_o\}$ necessarily responds to permission set \tilde{A}_t by choosing 0 with probability one. In other words, $a_t(\{p_o\}) = 1$ for each t . Now consider any type S with $\lim_{t \rightarrow \infty} a_t(S) < 1$. Along the equilibrium path $\{\tilde{A}_k\}_{k=0}^{\infty}$, type S eventually chooses 0 significantly less often than the singleton type $\{p_o\}$ in each period. Hence, after observing the Agent choosing 0 for a sufficient number of periods, the Principal eventually becomes quite certain that the Agent cannot be of type S . Thus, $\lim_{t \rightarrow \infty} \mu_t(S) = 0$. The formal proof proceeds as follows:

Proof. Recall that μ_t is the Principal's belief at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$ and $a_t(S)$ is the probability of type S choosing 0 when facing permission set \tilde{A}_t at history $\{\tilde{A}_0, \dots, \tilde{A}_{t-1}\}$. Hence, both μ_t and a_t are deterministic functions.

Take any type S such that $\lim_{t \rightarrow \infty} a_t(S) = 1$ is not true. There exists a subsequence $\{a_{t_k}(S)\}$ converging to a limit other than one— $\lim_{k \rightarrow \infty} a_{t_k}(S) = a_{\infty} < 1$. Hence, I can find T that

$$a_{t_k}(S) \leq \frac{1 + a_{\infty}}{2} \text{ for all } t_k \geq T. \quad (1)$$

Let Q_t denote the probability that the Agent chooses 0 facing all permission

sets $\tilde{A}_0, \dots, \tilde{A}_{t-1}$ in past periods. By Bayes' rule:

$$\mu_t(S) = \frac{\mu_0(S) \prod_{j=0}^{j=t-1} a_j(S)}{Q_t}.$$

Because $\cup_{k=0}^{k=\infty} \tilde{A}_k \neq N$, there exists singleton type $\{p_o\}$ with $p_o \notin \cup_{k=0}^{k=\infty} \tilde{A}_k$. By Bayes' rule:

$$\mu_t(\{p_o\}) = \frac{\mu_0(\{p_o\}) \prod_{j=0}^{j=t-1} a_j(\{p_o\})}{Q_t} = \frac{\mu_0(\{p_o\})}{Q_t}.$$

The second equality is because for all j , $a_j(\{p_o\}) = 1$ because $p_o \notin \tilde{A}_j$.

Consider the following likelihood ratio:

$$\frac{\mu_t(S)}{\mu_t(\{p_o\})} = \frac{\mu_0(S) \prod_{j=0}^{j=t-1} a_j(S)}{\mu_0(\{p_o\})} \leq \frac{\mu_0(S)}{\mu_0(\{p_o\})} \left[\frac{1 + a_\infty}{2} \right]^{|\{t_k | T \leq t_k < t\}|},$$

where the inequality is by [Equation \(1\)](#) and $a_j(S) \leq 1$.

Notice that $\frac{1 + a_\infty}{2} < 1$. Hence, $\lim_{t \rightarrow \infty} \frac{\mu_t(S)}{\mu_t(\{p_o\})} = 0$, which implies $\lim_{t \rightarrow \infty} \mu_t(S) = 0$. This completes the proof. \square

Proof of [Theorem 2](#)

Proof. In this static commitment setting, if the Principal permits all projects, she will receive the same payoff as in equilibrium. If the Principal permits all but her least preferred project, she will obtain a strictly higher payoff: Denote her least preferred project by \underline{p} . When the Principal permits all projects, each type S is going to choose project $\max S$. By removing project \underline{p} from the permission set, the Principal is going to obtain the same project if \underline{p} is not the Agent's favorite feasible project; the Principal is going to make a loss of $\pi_{\underline{p}}$ if the Agent

is of singleton type $\{\underline{p}\}$; the Principal's payoff will increase by $\pi_{\max S \setminus \{\underline{p}\}} - \pi_{\underline{p}}$ if the Agent is of type S other than singleton type $\{\underline{p}\}$ and that \underline{p} is exactly the Agent's favorite feasible project ($\max S = \underline{p}$). Hence, removing project \underline{p} will strictly increase the Principal's payoff if the total loss is smaller than the total increase in her payoff:

$$\mu_0(\{\underline{p}\})\pi_{\underline{p}} < \sum_{S: S \neq \{\underline{p}\}, \max S = \underline{p}} \mu_0(S)(\pi_{\max(S \setminus \{\underline{p}\})} - \pi_{\underline{p}}).$$

Because preferences are non-congruent, $\underline{p} \neq 1$. Agent type $\{1, \underline{p}\}$ will be in set $\{S : S \neq \{\underline{p}\}, \max S = \underline{p}\}$, and $\pi_1 - \pi_{\underline{p}} > 0$ because \underline{p} is the Principal's least preferred project and preferences are non-congruent. In addition, $\mu_0(\{1, \underline{p}\}) > 0$ because the prior has full support. Hence, the right-hand side of the above inequality is strictly positive. As long as the prior put sufficiently less probability on singleton $\{\underline{p}\}$, the above inequality will hold, and the Principal can achieve a strictly higher payoff in the static commitment setting than in equilibrium. \square

Proof of Theorem 3

Proof. Let T be a sufficiently large number such that no Agent type is willing to wait T periods to obtain a better project.

$$T \equiv \min\{t | \alpha_n \delta_A^t < \alpha_1\}.$$

The Principal commits to the following **Slowly Compromising Strategy**: Order the set of the Principal's payoff from potential projects ($\{\pi_p | p = 1, \dots, n\}$) in a decreasing manner $\pi_{(1)} > \dots > \pi_{(k)} > \dots > \pi_{(K)}$. In the first T pe-

riods, only permits projects with a payoff to the Principal at least $\pi_{(1)}$. For each $k = 1, \dots, K - 1$, in the k -th T periods (from period $(k - 1)T$ to period $kT - 1$), permits projects with payoffs to the Principal at least $\pi_{(k)}$. From period $(K - 1)T$ on, permits all projects. For each Agent type S , if the Principal's favorite project in set S gives her a payoff $\pi_{(k)}$ ($\max_{p \in S} \pi_p = \pi_{(k)}$), then under the Slowly Compromising Strategy, type S has no feasible project permitted before period $(k - 1)T$. In period $(k - 1)T$, all permitted projects in set S give the Principal payoff $\pi_{(k)}$; and by definition of T , the Agent has no incentive to wait for a better project. Hence, the Principal will receive payoff $\delta_P^{(k-1)T} \pi_{(k)}$ if the Agent has type S . Because T is independent of δ_P , this payoff will converge to $\pi_{(k)} = \max_{p \in S} \pi_p$ as δ_P goes to one. As a result, the Principal's payoff under the Slowly Compromising Strategy converges to the First Best Payoff as δ_P goes to one. □