Market segmentation through information

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Abstract

An information designer has information about consumers' preferences over products sold by oligopolists and chooses what information to reveal to firms who, then, compete on price by making personalized offers. We study the market outcomes the designer can achieve. The information designer is a metaphor for an internet platform which uses data on consumers to target advertisements that include discounts and promotions. Our analysis demonstrates the power that users' data can endow internet platforms with, and speaks directly to current regulatory debates.

1 Introduction

The last two decades have witnessed the emergence of a new internet based business model whereby revenue streams emanate from collecting and using information about users to target advertisements. Concerns about competition and users' privacy issues have attracted the attention of antitrust authorities around the world. We explore the power that information grants internet intermediaries in shaping market competition. We do so by extending the information design problem with a monopolist considered by Bergemann, Brooks, and Morris (2015) to an oligopoly setting.

Antitrust authorities often seem to lean on two benchmarks to guide their thinking towards possible economic harms in downstream markets—complete information, in which all firms know all consumers' preferences, and no information, in which all firms know only the distribution of consumers' preferences.[1] When firms have complete information, Bertrand competition leads to an efficient pricing equilibrium in which each consumer buys the product for which she is willing to pay the biggest premium above its marginal cost. In comparison, with no information on individual consumers' preferences, firms set prices to trade off the inframarginal losses on existing customers against attracting new customers, often leading on inefficient outcomes—with some consumers being inefficiently excluded from trading and others buying the wrong product from an efficiency perspective. Comparing these cases reveals that the use of information that permits price discrimination is typically welfare enhancing and can sometimes increase consumer

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[1] See, for example, (Council of Economic Advisors, 2015).

surplus.[2] This provides a salient, cautionary note for regulations that limit the use of information about consumer preferences. For instance, the Council of Economic Advisors (CEA) report on big data and price discrimination observes that "Economic reasoning suggests that differential pricing, whether online or offline, can benefit both buyers and sellers," and goes on to conclude that "we should be cautious about proposals to regulate online pricing." (Council of Economic Advisors, 2015).[3]

An intermediary who commands access to consumers' data has more options available than just choosing between either withholding all information, or disclosing all information to all firms. For example, in response to privacy concerns, Google has attempted to replace the use of third-party tracking cookies on its Chrome web browser with its "Privacy Sandbox". The "Privacy Sandbox" groups users into "cohorts" based on their browsing behaviour and targets firms' ads and promotions to these cohorts rather than to individuals. Technologies like this are able to package information about consumers and disclose it to firms in a fairly complicated way. The aim of this paper is to shed new light on the effect of such technologies on price competition.

We consider an information designer who chooses what information about consumers to reveal to competing firms who, then, play a simultaneous pricing game. The information designer can be thought of as an intermediary whose objective is increasing in consumer surplus and producer surplus; we study the maximal combinations of producer and consumer surplus such an intermediary can achieve by designing information structures. Two cases of particular interest are when the intermediary seeks to maximize either producer surplus or consumer surplus.

A simple example can illustrate the main ideas. Suppose there are two firms, A and B, both of which produce a single product and have a constant marginal cost of production set to 0. A single consumer has unit demand and her type is identified by her valuations for the products sold by each of the two firms (v_A, v_B). There is a mass 0.2 of consumers with types $(1, 0)$ and $(0, 1)$, a mass 0.2 with types $(4/5, 1/5)$ and $(1/5, 4/5)$ and a mass 0.1 with types $(3/5, 2/5)$ and $(2/5, 3/5)$. The total gains from trade in this economy are $S^* = 0.84$.

Consider a platform that perfectly knows the valuations of different consumers and can provide information to firms. Four benchmark information structures are illustrative.

Case 1: No information disclosure. The platform discloses no information to the firms. In equilibrium both firms charge a price of 4/5. Total producer surplus is roughly 76% of S^* and consumer surplus is a bit less than 10% of S^*. The outcome is inefficient as consumer types $(3/5, 2/5)$ and $(2/5, 3/5)$ do not trade.

Case 2: Full disclosure. The platform tells both firms the exact valuations of each consumers. In equilibrium consumers buy the product they value more highly, firm A sets prices $p_A(v_A, v_B) = \max\{v_A - v_B, 0\}$ and firm B sets prices $p_A(v_A, v_B) = \max\{v_B - v_A, 0\}$.

[2] While this debate been reopened by the possibility of data-driven price discrimination, its provenance dates back at least to Pigou (1920) and Robinson (1933).

[3] Likewise, the Digital Competition Experts Panel report writes: "There are many reasons why consumers may wish to share their data with a third party. This might enable them to access more accurate price information, to better compare goods and services or to access more tailored advice or recommendations. It may also help support a more effective market, for example where consumers can make a conscious choice to share their data in exchange for some benefit, for example a monetary payment, price discount or free service." (Digital Competition Expert Panel, 2019).

This outcome is efficient. The producer surplus is roughly 81% of S^* and the consumer surplus is 19%.

Case 3: Perfect market segmentation. We now show that the platform can disclose information to firms to induce an equilibrium where all available surplus is extracted by the firms. Firms receive message $m = (1, 3/5)$ if the consumer is either $(1, 0)$ or $(2/5, 3/5)$, message $m' = (4/5, 4/5)$ if the consumer is either $(4/5, 1/5)$ or $(1/5, 4/5)$ and a message $m'' = (3/5, 1)$ if the consumer is either $(3/5, 2/5)$ or $(0, 1)$. The messages are price recommendations for the two firms. For example, after receiving message $m = (1, 3/5)$, firm A sets a price 1 and firm B a price 3/5. It can be easily checked that no firm wants to deviate from these pricing recommendations.

The equilibrium outcome is efficient and all surplus S^* is extracted by producers. The market has been perfectly segmented through information. In this example, and in general, full market segmentation is obtained by grouping consumers who like product A the most with other consumers who value product A less and like another product more. Moreover, these sub-markets are constructed to incentivize each firm to engage in a niche market strategy and price to extract all surplus from the consumers who value its product the most, while excluding the other consumers.

In general, an information structure like this in which all possible surplus is extracted as producer surplus does not always exist. It may be necessary to give the firms different messages to achieve this outcome, and on other occasions private messages are not enough either. Theorem 1 provides a necessary and sufficient condition on the distribution of consumer valuations under which the information designer can induce each consumer to buy her most preferred product at a price equal to her valuation for it. This condition is easier to satisfy when consumers' preferences are more polarised, i.e., consumers have a strong taste for their most preferred product (Proposition 1). By contrast, when firms sell homogeneous products, there is no information structure which obtains any producer surplus (Proposition 2).

Case 4: Maximising price competition. Finally, we describe how the platform can provide information to firms to maximise consumer surplus. We consider the consumers with a higher value for A's product, a symmetric information structure holds for the consumers who value B's product the most. Consumers who value product A the most are placed in two groups and each group is assigned a message. The first group, which is assigned the message $m = (3/5, 0)$, contains a mass 0.175 of the $(1, 0)$ consumers and a mass 0.175 of $(4/5, 1/5)$ consumers, the second group, with message $m' = (1/5, 0)$, contains a mass 0.025 of $(1, 0)$ consumers a mass 0.025 of $(4/5, 1/5)$ consumers and all the $(3/5, 2/5)$ consumers. Again the messages are price recommendations and it can be easily checked that both firms are incentivized to follow them.

The equilibrium outcome is again efficient, but now the producer surplus is roughly 57% of S^* and the consumer surplus is roughly 43% of S^*. This is the equilibrium with the largest consumer surplus across all equilibria that can be induced by any information structure. To see this, consider an arbitrary information structure. An option available to firm 1 is to ignore the signals it receives and to set the same price to all consumers. Pursuing this strategy, the worst case scenario for firm 1 is the one in which 2 sets a price of 0 to all consumers. In this case, the profit maximizing uniform price for firm 1 is a price of 0.6 yielding it profits equal to roughly 28.5% of S^*. This is a lower bound on the profits that firm 1 can guarantee itself. By symmetry, the same lower bound applies for firm 2, yielding an overall lower bound on producer surplus of roughly 57% of S^*. As the information structure proposed above achieves this bound, and all remaining surplus goes to consumers, consumer surplus is maximized. This result is generalized in Theorem 2 which extends to price competition the consumer optimal information structure for the monopoly problem in Bergemann, Brooks, and Morris (2015).

In this example, and in general, the consumer surplus maximizing outcome is obtained by grouping together only consumers who like the same product the most, and then incentivising the producer of this product to set a price that all such consumers are willing to pay—i.e., firms are incentivised to play a mass market strategy.

Observe that both the consumer-optimal and producer-optimal outcomes are efficient—in both cases all consumers buy the product they value most. We also consider information structures that achieve the other points along the efficient frontier and provide a sufficient condition under which all interior points of the frontier can be achieved.

A comparison of the producer-optimal and consumer-optimal structure is of particular interest to antitrust authorities mandated to protect consumer surplus. First, both the producer-optimal and consumer-optimal information structures are consistent with privacy enhancing technologies like Google's Privacy Sandbox: they both pool consumers into flocks and transmit this coarsened information to competing firms. Hence, an intermediary which monetizes the firms' side may have strong incentives to develop privacy enhancing technologies that create groups like those in the producer-optimal information structure. In this case, enhancing users' privacy in this way is no impediment to extracting consumer surplus—to the contrary, it facilitates it. Our analysis reveals that the producer-optimal and consumer-optimal information structures are constructed based on distinctive principles and therefore can potentially help aid regulation. For example, regulators might formulate guidelines or rules of conduct which ensure that such groups of consumers are formed in line with the principles characterizing the consumer-optimal structure: only consumers with similar preferences (and hence the same most preferred product) should be grouped together and information should be disclosed symmetrically across firms.[4]

1.1 Related literature. Our paper contributes to a recent literature studying how information shape consumer and producer surplus. Bergemann, Brooks, and Morris (2015) characterizes the consumer and producer surplus outcomes attainable when a designer can provide different information on consumer valuations to a monopolist able to price discriminate. We extend the analysis to an oligopoly setting—the introduction of competition poses additional technical challenges, but also leads to new economic insights which can be related to contemporary regulatory debates.[5]

[4]For instance, the regulators can formulate rules of conduct prescribing that machine learning techniques used to aggregate consumers in flocks to have the objective to group consumers with similar values together, as in the consumer-optimal structure. For related issues about algorithms used directly by competing firms interacting with each other and softening competition (possibly inadvertently) see Calvano et al. (2020).

[5]A large literature has studied how firms choose which information about an aggregate parameter—for instance, about a demand or cost shock—to share when competing, see, among others, Novshek and Sonnenschein (1982), Vives (1988), Raith (1996). Several recent papers have taken a design approach

Bergemann, Brooks, and Morris (2017) study an information design problem in a first price auction with correlated bidders' value. The information designer discloses to bidders information on their own and others' value. They characterize the revenue minimizing (bidder surplus maximizing) information structure when the prior distribution of value is symmetric. Both our papers investigate an information design problem in which information is disclosed to economic agents who compete with each others (bidders compete for the one unit of good *viz.* firms compete for the unit demand of consumers). However, the two settings are not isomorphic and, in fact, the methods employed for the analysis as well as the economic insights obtained are distinctive and complementary.[6] For example, in Bergemann, Brooks, and Morris (2017) there is no information structure which implements the bidders' collusive outcome. In contrast, we show that under "enough product differentiation" the market can be segmented through information to implement the firms' collusive outcome. On the other hand, when products are homogeneous, we show that price competition between firms drive all firms' profit to zero under any information structure. In contrast, when bidders have the same value (common value auction), Bergemann, Brooks, and Morris (2017) show there exist information structures implementing outcomes with positive surplus to bidders.

We investigate what outcomes an intermediary with exogenous consumer data can achieve by sharing the data with firms. Complementary to this, Ali, Lewis, and Vasserman (2020) consider a disclosure game in which a consumer chooses some verifiable information about her preferences to convey to firms. They show that the ability to reveal only partial information can play firms against each other and intensify competition. We focus on a setting in which firms are uncertain about consumer valuations, while Roesler and Szentes (2017) study the converse problem in which consumers have uncertain valuation and face a monopolist which prices uniformly; they characterise the signal structure which is best for consumers. Armstrong and Zhou (2019) extend this setting to the duopoly case with uniform pricing, and characterise both firm-optimal and consumer-optimal signal structures.

Our paper also relates to a burgeoning literature on markets for information broadly conceived—the transaction, pricing, and design of information (see, e.g., Admati and Pfleiderer (1986), Armstrong and Vickers (2019) Lizzeri (1999), Taylor (2004) Calzolari and Pavan (2006), Bergemann and Bonatti (2015), Bergemann et al. (2018), Acemoglu et al. (2019), Bergemann et al. (2019), Fainmesser and Galeotti (2019), Kehoe et al. (2018) Montes et al. (2019), Jones and Tonetti (2020), Bounie, Dubus, and Waelbroeck (2020); also see Bergemann and Bonatti (2019) for a summary). Perhaps the closest paper to ours is Bounie, Dubus, and Waelbroeck (2018). Like us, they consider an intermediary choosing what information to reveal to firms about consumer valuations. A major focus in their paper is when the intermediary will share information to a single firm or both.

and studied how equilibrium varies in the information structure (Bergemann and Morris, 2013; Bimpikis, Crapis, and Tahbaz-Salehi, 2019). We study a setting with heterogeneous valuations and consumer-specific pricing, as well as explicitly model an information designer who has granular information at the level of individual valuations and chooses what information to reveal to each firm.

[6]Recasting our framework as an auction would not be natural because consumers have heterogeneous preferences over products offered by different firms and this implies that the allocation rule (determining the firm a consumer will buy from) cannot only depend on the profile of bids (the posted prices), but it must also depend on which firm sets which bid. In addition, in the auction studied by Bergemann, Brooks, and Morris (2017), there is a winner's curse, since bidders' values are correlated. In contrast, in our setting firms face no additional uncertainty about their profit conditioning on making a sale.

They conduct their analysis within a Hotelling model with linear transportation costs and restrict the set of possible information structures that the intermediary can offer firms. We abstract away from the way industry profits are shared between firms and the intermediary and study our information design problem in a general oligopoly model with differentiated products and arbitrary information structures.

Finally, from a technical point of view, we make use of the well-known interpretation of information design as a problem of matching (i.e., looking for a joint distribution over actions and states) which fulfils (i) incentive compatibility; and (ii) a martingale constraint that the marginal over states must equal the prior. Several recent papers take this view and make progress on nonlinear persuasion for a single receiver (Kolotilin, Corrao, and Wolitzky, 2022; Dworczak and Kolotilin, 2022) Our setting works with a multidimensional state (the consumer's valuation for each good) with multiple receivers and the designer is interested in the joint distribution of the state and the full profile of receivers' actions.

2 Model

There is a finite set of firms, indexed $\mathcal{N} = \{1, \ldots, n\}$ each of which produces a single product at zero cost. There is a continuum of consumers with unit mass each of whom demands a single unit inelastically.[7] A consumer of type $\boldsymbol{\theta} := (\theta_1, \theta_2, \ldots, \theta_n)$ obtains utility $\theta_i \in V$ from purchasing from firm i where $V = \{v_1, \ldots, v_K\}$, and $\{0 < v_1 < v_2 < \ldots < v_K < 1\}$, $K < \infty$. The distribution of consumers over V^n is given by the mass function $f : V^n \to [0, 1]$ such that $\sum_{\boldsymbol{\theta} \in V^n} f(\boldsymbol{\theta}) = 1$, which is common knowledge. We will primarily work with the support of the distribution, which we denote with $\Theta := \text{supp} f \subseteq V^n$.

We denote the consumer types that value product i the most by $E_i := \{\boldsymbol{\theta} \in \Theta : \theta_i > \theta_j \text{ for all } j \neq i\}$. We assume that all consumers have strict preferences so that there is no mass on the types $\{\boldsymbol{\theta} \in V^n : |\operatorname{argmax}_j \theta_j| > 1\}$. This implies that $\{E_i\}_{i \in \mathcal{N}}$ partitions Θ. We focus on discrete type distributions and assume preferences are strict just to streamline the exposition. All our main results extend to a continuous version of the model which we develop in Online Appendix B.2.

An information designer, knowing the valuation of each consumer for each product, commits to an information structure which, for each consumer type, specifies a distribution over messages each firm receives. Thus, the information designer chooses a mapping

$$\boldsymbol{\psi} : \Theta \to \Delta(\mathcal{M})$$

from consumer types to a joint probability distribution over messages $\Delta(\mathcal{M})$ where $\mathcal{M} = \prod_{i \in \mathcal{N}} M_i$, and $M_i = [0, 1]$ is the message space for firm i. Denote the set of information structures with $\boldsymbol{\Psi}$ and for $\boldsymbol{\psi} \in \boldsymbol{\Psi}$, $\psi_i(\boldsymbol{\theta})$ is the marginal distribution of messages firm i receives.

Call $m_i \in M_i$ a message realisation for firm i. Given the messages received, firms play a simultaneous move pricing game. A pure strategy for firm $i \in \{1, \ldots, n\}$ is $p_i : M_i \to [0, 1]$. A mixed strategy for firm i is $\sigma_i : M_i \to \Delta([0, 1])$. Each consumer then observes

[7] All results translate into an alternate setting with a single consumer of uncertain type.

the prices she is being offered by the different firms and chooses to either (i) purchase a product which maximizes her surplus given these prices; or (ii) not purchase any product and obtain zero surplus.

The information designer can be thought of as an intermediary that has detailed information about consumer preferences, and chooses what market segmentation of consumers to present to each firm.[8]

3 Producer-Optimal Information Structure

We first characterize conditions under which there exist information structures such that, in an equilibrium of the resultant subgame, the following property holds:

P **(Full Surplus Extraction)** each consumer of type $\boldsymbol{\theta} \in \Theta$ pays $\max_{i \in \mathcal{N}} \theta_i$.

Condition P characterizes the firms' fully collusive outcome (joint surplus maximizing outcome) when transfers are possible; an equilibrium that satisfies conditions P is efficient in so far as no surplus is left on the table. Let $\Gamma(\boldsymbol{\psi})$ denote the pricing subgame induced by the information structure $\boldsymbol{\psi}$. Let $\boldsymbol{\Gamma}^*$ denote the set of induced games in which there exists an equilibrium satisfying condition P, and let $\boldsymbol{\Psi}^* := \{\boldsymbol{\psi} : \Gamma(\boldsymbol{\psi}) \in \boldsymbol{\Gamma}^*\}$ be the set of information structures that can be used to fulfil condition P. We refer to $\boldsymbol{\psi} \in \boldsymbol{\Psi}^*$ as a producer-optimal information structure and to the induced outcome as the producer-optimal outcome.[9] We say that a producer-optimal information structure exists whenever $\boldsymbol{\Psi}^* \neq \emptyset$.

Suppose an information structure induces a producer-optimal outcome. Then consumers of type $\theta \in E_i$ must buy from firm i at a price $p_i = \theta_i$. A possible deviation available to firm i is to then deviate downwards and set a price $\hat{p}_i < \theta_i$ to all consumers types $\theta \in E_i$ such that $\theta_i > \hat{p}_i$. At this lower price firm i will continue to sell to all these consumers and might be able to make some additional sales to consumer types $\theta' \notin E_i$. Indeed, there is an upper bound on the additional sales firm i can possibly make via such a deviation. At best, firm i can make additional sales to all those consumer types $\theta' \notin E_i$ who value i's product weakly above $\hat{p}_i$. Thus a sufficient condition for firm i to not want to deviate downwards in this way is

$$\sum_{\theta_i > \hat{p}_i} (\theta_i - \hat{p}_i) \sum_{\substack{\boldsymbol{\theta}' \in E_i: \\ \theta'_i = \theta_i}} f(\boldsymbol{\theta}') \geq \hat{p}_i \sum_{\substack{\boldsymbol{\theta}' \in \Theta \setminus E_i: \\ \theta'_i \geq \hat{p}_i}} f(\boldsymbol{\theta}'), \tag{AIC}$$

[8] Bergemann and Morris (2013, 2016) consider many-player settings and examine how the informational environment maps to resultant equilibria. In the special case with a single receiver, Kamenica and Gentzkow (2011) show that concavification of the designer's payoff as a function of receiver's posteriors binds the designer's maximum attainable utility and characterises the optimal signal structure (see also Kamenica (2019)). However, there are well-known difficulties applying such techniques when the type space is large. A contribution of our analysis is to show that it can be helpful to reframe certain information design problems as matching problems (see also Kolotilin, Corrao, and Wolitzky (2022) who study settings with a single receiver).

[9] This terminology is slightly non-standard. In the literature the term producer-optimal outcome is often used to refer to the maximum amount of producer surplus that can be obtained with an information design.

where AIC abbreviates aggregate incentive compatibility for reasons which will soon be apparent. The left-hand side of this inequality is firm i's aggregate infra-marginal losses from setting price $\hat{p}_i \leq \theta_i$ instead of θ_i to all consumers in E_i with valuations above $\hat{p}_i$, and the right-hand side is the maximum business stealing profit that firm i can hope to obtain from such a deviation.

It is not obvious that condition AIC needs to be satisfied in a producer-optimal design, or that satisfying it is sufficient to achieve the producer optimal outcome—it only considers some very particular deviations and, for those deviations, it may be overly optimistic about the profitability of them.

Theorem 1. A producer-optimal information structure exists if and only if for all firms $i \in \mathcal{N}$ and all $\hat{p}_i \in V$ the aggregate incentive compatibility condition (AIC) holds.

We first outline the proof of Theorem 1, then present an example to illustrate it and finally we come back to discuss its implications.

3.1 Proof outline. The first steps towards proving Theorem 1 are to show that the condition P dramatically simplifies the space of possible information structure; An information design that induces an equilibrium satisfying property P must satisfy several conditions.

In order to satisfy condition P all consumers must buy their most preferred product and pay their full valuation for it. This implies that messages firm i receives must perfectly separate consumers $\boldsymbol{\theta}, \boldsymbol{\theta}' \in E_i$ with different values $\theta_i \neq \theta_i'$ for product i. Formally, let $\text{supp}\big(\psi_i(\boldsymbol{\theta})\big)$ be the support of $\psi_i(\boldsymbol{\theta})$ (the probability distribution over the messages i can receive for a given type $\boldsymbol{\theta}$). We obtain: $\boldsymbol{\psi} \in \boldsymbol{\Psi}^*$ only if for all $i \in \mathcal{N}$:

$$\text{supp}\big(\psi_i(\boldsymbol{\theta})\big) \cap \text{supp}\big(\psi_i(\boldsymbol{\theta}')\big) = \emptyset \text{ for all } \boldsymbol{\theta}, \boldsymbol{\theta}' \in E_i \text{ such that } \theta_i \neq \theta_i'. \qquad \textbf{(Separation)}$$

Under a producer optimal information structure firm i must be unable to separate a consumer in E_j from all the consumers in E_i. Indeed, suppose that firm i learns that a consumer belongs to E_j. Because firm j extracts the consumer valuation for product j, firm i can target a price $p_i \in (0, v_1)$ to the consumer, who will buy product i. This violates condition P. Formally, $\boldsymbol{\psi} \in \boldsymbol{\Psi}^*$ only if for all $i \in \mathcal{N}$:

$$\bigcup_{\boldsymbol{\theta} \in E_i} \text{supp}\big(\psi_i(\boldsymbol{\theta})\big) \supseteq \bigcup_{\boldsymbol{\theta} \notin E_i} \text{supp}\big(\psi_i(\boldsymbol{\theta})\big) \qquad \textbf{(Consistency)}$$

Separation and **Consistency** imply that, when characterizing producer-optimal information structure, we can restrict our attention to information designs where the set of messages that firm i receives is the set of valuations for product i of consumers in E_i, denoted by $M_i' := \text{supp}\{\theta_i : \boldsymbol{\theta} \in E_i\} \subseteq V$, and messages are price recommendations.

Lemma 1. A producer-optimal information design exists when the set of available messages is $\mathcal{M}$ if and only if a producer-optimal information design exists when the set of available messages is $\mathcal{M}' := \prod_{i \in \mathcal{N}} M_i'$.

From now on, we focus on information structures that satisfy **Separation** and **Consistency**, and restrict the message space to $\mathcal{M}'$ and messages are price recommendations.[10] Hence, for a producer-optimal information design we need to define, for each firm i, how to assign consumers not in E_i to price recommendations M_i' in a way that firm i follows the price recommendations (**Firm IC**) and consumers in E_i buys product i (**Consumer IC**).

We start with **Consumer IC**. Consider a consumer of type $\boldsymbol{\theta} \in E_j$ and suppose all firms follow the price recommendations. Firm j charges θ_j to this consumer. If firm i receives message m_i about this consumer, the consumer can buy product i at a price m_i. **Consumer IC** implies that the consumer valuation θ_i must be lower than m_i. Formally, $\boldsymbol{\psi} \in \boldsymbol{\Psi^*}$ only if for all $\boldsymbol{\theta} \notin E_i$,

$$\psi_i(m_i|\boldsymbol{\theta}) = 0 \text{ for all } \theta_i \geq m_i. \qquad \textbf{(Consumer IC)}$$

We finally consider **Firm IC**. **Consumer IC** implies that a firm never wishes to charge a price above the price recommendation (as the demand will be zero). Hence, we only need to prevent that, upon receiving message m_i, undercutting deviations $\hat{p}_i < m_i$ are not profitable: the infra-marginal losses for consumers in E_i (now being charged a price less than their valuations) must be greater than the extra profits made via any additional sales to consumers not in E_i. Formally, $\boldsymbol{\psi} \in \boldsymbol{\Psi^*}$ only if for all $m_i \in M_i'$ and for all $\hat{p}_i < m_i$,

$$\underbrace{(m_i - \hat{p}_i) \sum_{\boldsymbol{\theta'} \in E_i : \theta_i' = m_i} f(\boldsymbol{\theta'})}_{\text{Inframarginal losses}} \geq \underbrace{\hat{p}_i \sum_{\boldsymbol{\theta'} \in \Theta \setminus E_i : \theta_i' \geq \hat{p}_i} \psi(m_i|\boldsymbol{\theta'}) f(\boldsymbol{\theta'})}_{\text{Business stealing gains}}. \qquad \textbf{(Firm IC)}$$

Lemma 2 summarizes the properties of a producer-optimal information structure.

Lemma 2. A producer-optimal information structure exists if and only if there exists an information structure $\boldsymbol{\psi}$ which, for all firms $i \in \mathcal{N}$, satisfies **Separation**, **Consistency**, **Consumer IC** and **Firm IC**.

Let $\boldsymbol{\psi}$ satisfy **Separation**, **Consistency** and **Consumer IC**. The next step in proving Theorem 1 is to determine the maximum mass of types not in E_i that can be matched to each of firm i's message $m_i \in M_i'$ without violating one of firm i's incentive compatibility conditions. Thus, we consider the mass of types not in E_i that can be assigned to a given message m_i for firm i that makes firm i indifferent between following the recommendation and deviating to *any* price $\hat{p}_i \leq m_i$. For all $m_i \in E_i$, this matching capacity is given by

$$\sum_{\boldsymbol{\theta'} \in \Theta \setminus E_i : \theta_i' \geq \hat{p}_i} \psi(m_i|\boldsymbol{\theta'}) f(\boldsymbol{\theta'}) = \frac{(m_i - \hat{p}_i)}{\hat{p}_i} \sum_{\boldsymbol{\theta'} \in E_i : \theta_i' = m_i} f(\boldsymbol{\theta'}) \quad \text{for all } \hat{p}_i \leq m_i.$$

The right-hand-side is the exact mass of consumers not in E_i with valuation for i's product in $[\hat{p}_i, m_i)$ that, if matched to m_i, makes firm i indifferent between charging m_i

[10] We can therefore rewrite Separation and Consistency as follows: Separation for all $\boldsymbol{\theta} \in E_i$ and for all firms i, $\psi_i(m_i|\boldsymbol{\theta}) = 1$ if $m_i = \theta_i$ and $\psi_i(m_i|\boldsymbol{\theta}) = 0$ otherwise; Consistency: for all $\boldsymbol{\theta} \notin E_i$ and for all firms i, $\sum_{m_i \in M_i'} \psi_i(m_i|\boldsymbol{\theta}) = 1$.

and $\hat{p}_i$. In other words, the matching capacity for each message m_i and each deviation $\hat{p}_i < m_i$. We define this as:

$$G_i(\hat{p}_i, m_i) := \frac{(m_i - \hat{p}_i)}{\hat{p}_i} \sum_{\boldsymbol{\theta}' \in E_i : \theta'_i = m_i} f(\boldsymbol{\theta}').$$

To find the overall capacity for matching consumer types $\theta \notin E_i$ to messages M'_i we sum the $G_i(\hat{p}_i, m_i)$ across all possible price recommendations greater than $\hat{p}_i$ that firm i can receive. Formally, we define the function:

$$H_i(\hat{p}_i) := \sum_{m_i > \hat{p}_i} G_i(\hat{p}_i, m_i).$$

The value of $H_i(\hat{p}_i)$ gives us a maximum mass of types not in E_i with a value for product i larger than $\hat{p}_i$ that can be matched to messages $m_i > \hat{p}_i$ if we wish to bind all firm i's IC constraints. But the available mass of consumers not in E_i which have at least value $\hat{p}_i$ for i's product is

$$\sum_{\boldsymbol{\theta}' \in \Theta \setminus E_i : \theta'_i \geq \hat{p}_i} f(\boldsymbol{\theta}').$$

Hence, if this mass of consumers is greater than $H_i(\hat{p}_i)$ then we cannot construct a producer-optimal structure: there is no way to assign all these consumers messages $m_i \in \{M'_i : m_i > \hat{p}_i\}$ without firm i sometimes having a profitable deviation to capture some of these consumers. On the other hand, if this mass of consumers is weakly less than $H_i(\hat{p}_i)$ for all $\hat{p}_i$, then there is a way of assigning the consumers not in E_i messages $m_i \in V$ such that firm i wants to follow the price recommendation m_i. These steps, as well as the prior ones, are formalized in Appendix A.

We have shown that an information structure exists that induces an equilibrium satisfying condition P if and only if for all firms i and all prices $\hat{p}_i$,

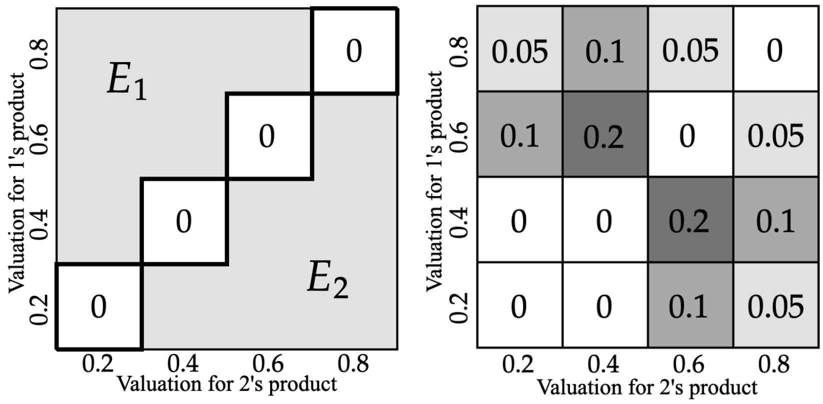
$$H_i(\hat{p}_i) \geq \sum_{\boldsymbol{\theta}' \in \Theta \setminus E_i : \theta'_i \geq \hat{p}_i} f(\boldsymbol{\theta}'),$$

but this is just the AIC condition.

3.2 Producer-optimal information structure: an example. The example illustrates how to check existence and features of a producer-optimal information structure. A reader familiar with information design can skip this example without loss. There are two firms, 1 and 2; Panel (b) of Figure 1 summarizes the mass of consumers of each type and Panel (a) identifies the consumer types in E_1 and E_2.

By following price recommendation $m_1 = 0.8$, firm 1 sells to consumers in E_1 with $v_1 = 0.8$ and makes a profit of 0.16. Firm 1 obtains the same profit by deviating to a price of $\hat{p}_1 = 0.6$ when the mass of consumers in E_2 with $v_1 \in [0.6, 0.8)$ that is matched to $m_1 = 0.8$ is $G_1(0.6, 0.8) = 0.2/3$. Similarly, a deviation to a price $\hat{p}_1 = 0.4$ will generate a profit of 0.2 when the mass of consumers in E_2 with $v_1 \in [0.4, 0.8)$ matched to $m_1 = 0.8$ is $G_1(0.4, 0.8) = 0.2$. Finally, to make firm 1 indifferent about deviating to $\hat{p}_1 = 0.2$ we must have $G_1(0.2, 0.8) = 0.6$.

Figure 1: The set E_1 and E_2, and distribution of consumers types.

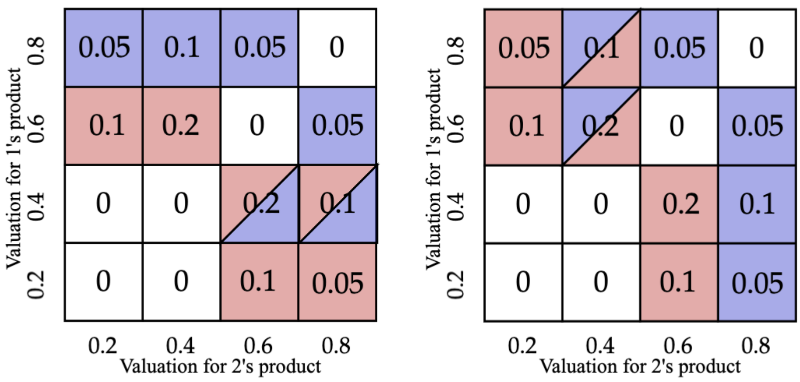


Next, consider price recommendation $m_1 = 0.6$. If firm 1 follows the recommendation, it will sell to a 0.3 mass of consumer in E_1 obtaining a profit of 0.18. Note that by firm 1's IC constraint none of the 0.05 mass of consumers in E_2 with $v_1 = 0.6$ can be assigned to this message—otherwise firm 1 can slightly lower the price 0.6 and increase profits. Firm 1 is indifferent about deviating to $\hat{p}_1 = 0.4$ when an additional mass of $G_1(0.4, 0.6) = 0.15$ consumers in E_2 with $v_1 \in [0.4, 0.6)$ are matched to message $m_1 = 0.6$. Likewise, for firm 1 to be indifferent about deviating to $\hat{p}_1 = 0.2$, it must gain a mass of consumers in E_2 equal to $G_1(0.2, 0.6) = 0.6$.

The condition in the Theorem 1 is satisfied for firm 1 (and analogously for firm 2) because

$$\begin{aligned} H_1(0.2) &= 1.2 > f(0.2,0.6) + f(0.2,0.8) + f(0.4,0.6) + f(0.4,0.8) + f(0.6,0.8) = 0.5 \\ H_1(0.4) &= 0.35 = f(0.4,0.6) + f(0.4,0.8) + f(0.6,0.8) = 0.35 \\ H_1(0.6) &= 0.2/3 > f(0.6,0.8) = 0.05 \end{aligned}$$

Figure 2: A producer-optimal information structure.
Note: the proportion of each square shaded blue and red denotes the proportion of each type assigned the messages 0.8 and 0.6 provided to each firm respectively.



(a) Information for firm 1 (b) Information for firm 2

Figure 2 illustrates a producer-optimal information structure. In Panel (a), the proportion of each square shaded blue and red denotes the proportion of each type assigned the price recommendations 0.8 and 0.6 that firm 1 receives; Panel (b) does the same for firm 2. Upon receiving message 0.6 firm 1 learns this group contains 0.3 mass of consumers in E_1 with $v_1 = 0.6$, 0.15 mass of consumers in E_2 with $v_1 = 0.4$, and 0.15 mass of consumers in E_2 with $v_1 = 0.2$. Firm 1 is indifferent between charging 0.6 and 0.4 and strictly prefers to charge 0.6 instead of 0.2. Upon receiving message 0.8 firm 1 learns that this group contains all 0.2 mass of consumers in E_1 with $v_1 = 0.8$, a mass of 0.05 consumers in E_2 with $v_1 = 0.6$, and a mass of 0.15 of consumers in E_2 with $v_1 = 0.4$. With this information, firm 1 strictly prefers to charge 0.8 instead of charging 0.6 or 0.2 and is indifferent between charging 0.8 and 0.4.

3.3 Consumers' preferences polarization. We now investigate when the condition in Theorem 1 is more likely to be satisfied.

In online Appendix B.3 we consider some canonical distributions in the continuous version of our model. A first benchmark in which a producer-optimal outcome exists is when consumers' valuations are uniformly distributed over the unit square. A producer-optimal outcome also exists in the Hotelling duopoly setting (anti-correlated values) when the valuations for the two products (v_1, v_2) are such that $v_2 = 1 - v_1$ and v_1 is uniformly distributed over the unit interval. When, instead, v_1 is drawn from a truncated (at zero and 1) normal distribution with mean 1/2 and variance σ^2, the producer-optimal outcome is feasible for all $\sigma > 0.15$. Intuitively, as the variance increases, consumers have stronger preferences for one product over the other and this slackens constraints posed by **Firm IC** which, in turn, strengthens the designer's ability to implement a producer-optimal information structure. We now show that this holds generally.

Definition 1. Consumers' preferences are more polarized under distribution $\tilde{f}$ relative to distribution f whenever

(i) The mass of consumers who, under f, prefer i's product and have valuations greater than $\hat{\theta}_i$ for it must increase under $\tilde{f}$ i.e., for all $i \in \mathcal{N}$,

$$\sum_{\boldsymbol{\theta}' \in E_i : \theta'_i \geq \hat{\theta}_i} f(\boldsymbol{\theta}') \leq \sum_{\boldsymbol{\theta}' \in E_i : \theta'_i \geq \hat{\theta}_i} \tilde{f}(\boldsymbol{\theta}') \quad \text{for all } \hat{\theta}_i \in V,$$

(ii) The mass of consumers who, under f, prefer j's product but have valuations greater than $\hat{\theta}_i$ for i's product must decrease under $\tilde{f}$ i.e., for all $i, j \in \mathcal{N}$,

$$\sum_{\boldsymbol{\theta}' \in E_j : \theta'_i \geq \hat{\theta}_i} f(\boldsymbol{\theta}') \geq \sum_{\boldsymbol{\theta}' \in E_j : \theta'_i \geq \hat{\theta}_i} \tilde{f}(\boldsymbol{\theta}') \quad \text{for all } \hat{\theta}_i \in V.$$

Proposition 1 (Polarization aids segmentation)**.** Assume consumers' preferences are more polarized under $\tilde{f}$ relative to f. If a producer-optimal information structure exists under f then it also exists under $\tilde{f}$, i.e., $\mathbf{\Psi^*} \neq \emptyset$ under f then $\mathbf{\Psi^*} \neq \emptyset$ under $\tilde{f}$.

There are various ways in which consumers' preference can become more polarised. An obvious avenue is for firms to make their products more differentiated.[11] We refer to Johnson and Myatt (2006) for many other examples on how firms can use product design and advertising to shape the distribution of consumers' preferences. As these managerial options increase polarization they also aid the feasibility of the producer-optimal outcome.

On the other hand, firm actions that uniformly increase the value consumers place on one product relative to another, thereby skewing the mass of consumer valuations towards a particular firm can inhibit the ability to achieve the producer-optimal outcome. This is because the firm with a reduced consumer base has stronger incentives to undercut other firms. Hence, imbalanced competition in which some firms have a much smaller market share than others can severely inhibit an intermediary from implementing a producer-optimal information structure.[12]

Furthermore, it can also be seen that a merger weakens the condition in Theorem 1. Consider a merger between two firms i and j, with $E_i \neq \emptyset$ and $E_j \neq \emptyset$, into the firm k (with the same product offering). The new condition that must be satisfied for the producer optimal outcome will be weaker than either of the two conditions required prior to the merger. This is because $E_k \supset E_i$ and $E_k \supset E_j$ which slackens the condition under which the producer optimal outcome is feasible for the merged firm, without affecting the conditions for the other firms.

Finally, we have thus far focused on the benchmark case in which the only role of the intermediary is to provide information about consumer preferences. In practice, however, the intermediary might, in addition to providing information about preferences, also control what price offers each consumer can evaluate by withholding firms' access to certain consumers, see Bergemann and Bonatti (2019) for a discussion on the distinction between information and access design. We show that, if in addition to designing information, the intermediary can restrict some firms' access to certain consumers, then the intermediary can do so by, de-facto, increasing consumers' preference polarization, thereby weakening the conditions under which the producer-optimal outcome can be achieved. In this sense, access to consumers is complementary to information design.[13]

3.4 The necessacity of product differentiation. It is easy to see that condition in Theorem 1 will fail when firms offer products which are highly substitutable.[14] The extreme case is one where firms offer homogeneous products. That is, each consumer type $\boldsymbol{\theta}$ has the same valuation across products $\theta_i = \theta_j$ for all products i and j, but some types

[11] This includes vertical differentiation when the marginal cost of production is increasing in the product quality. Consider two firms producing products at quality $q_1 \in (1/2, 1)$ and $q_2 = 1 - q_1$. Consumer γ's valuation for product i is $\alpha + \gamma q_i$ where $\alpha > 0$ and γ is drawn from a distribution with support $[0, 2]$. It is easy to check that when $q_1' > q_1$ consumers preferences are more polarized.

[12] This is similar to the argument that, in the presence of switching costs, firms with a smaller customer base can be a stronger competitive constraint on market behaviour than more established firms (Klemperer, 1995). Based in part on this logic the UK antitrust authorities prohibited the acquisition of Abbey National by Lloyds TSB Group in 2001.

[13] We formalize these claims in Proposition 4 in Online Appendix B.1. In a companion paper (Elliott, Galeotti, Koh, and Li, 2022) we consider a platform with joint control over access and information and characterize all feasible welfare outcomes it can implement.

[14] There exists $\epsilon > 0$ such that if, for each $\boldsymbol{\theta} \in \Theta$, $|\theta_i - \theta_j| < \epsilon$ for all products i and j, then the producer-optimal outcome is not feasible

may have high valuation than others.[15] How does information about the distribution of consumer valuations matter for price competition under homogenous products?

Proposition 2. Suppose firms offer homogeneous products, i.e., $\text{supp} f \subseteq \text{diag} V^n$. Then for any $\boldsymbol{\psi} \in \boldsymbol{\Psi}$ and any equilibrium induced by $\boldsymbol{\psi}$, (i) each consumer buys from some firm at a price of zero; and (ii) all firms make zero profits.

Intuitively, when firms have no further information beyond their common prior f, standard undercutting arguments imply producer surplus must be zero and consumers are charged 0. If all firms observed a common public signal prior to making the pricing decision then, by the same logic applied to the posterior distribution induced by each message realization, no consumer can be charged a strictly positive price in equilibrium. Proposition 2 extends this to the case where the designer can send private signals to firms.

4 Characterization of Consumer-Optimal Information Structure

We first establish an upper bound on consumer surplus. We then construct an information structure that implements it. Let $\boldsymbol{P}_{-i} \in \Delta(\Theta \times [0,1]^{n-1})$ denote the joint distribution of types and firms' prices (other than firm i). Facing $\boldsymbol{P}_{-i} \in \Delta(\Theta \times [0,1]^{n-1})$, firm i can always ignore any information received by the designer, select an optimal uniform price and obtain $\Pi_i^*(\boldsymbol{P}_{-i})$. This profit is minimized when all other firms charge all consumers a price of zero and so this corresponds to a lower bound on firm i's profits in any equilibrium. This lower bound is simply:[16]

$$\underline{\Pi_i^*} = \max_{p_i} p_i \sum_{\substack{\boldsymbol{\theta} \in E_i : \theta_i - p_i \geq \theta_j \\ \text{for all } j \neq i}} f(\boldsymbol{\theta}).$$

A corresponding upper bound on consumer surplus in all equilibria is, therefore,

$$CS^* = \sum_{i=1}^{n} \sum_{\boldsymbol{\theta} \in E_i} f(\boldsymbol{\theta}) \theta_i - \sum_{i=1}^{n} \underline{\Pi_i^*},$$

where the first part is the total surplus available in the economy.

We now show that there exists some information structure that always induces this welfare outcome in the resultant subgame. Suppose the designer publicly sends n messages $\{m_1, m_2, .., m_n\}$. Upon receiving m_i all firms learn that the consumers associated to m_i are all types in E_i. Further suppose that, given message m_i, all firms $j \neq i$ offer their

[15] In a duopoly this corresponds to all types be concentrated in the diagonal of the valuation grid. Note that we are now relaxing the assumption that consumers always have strict preferences.

[16] Facing $\boldsymbol{P}_{-i}$, firm i makes

$$\Pi_i^*(\boldsymbol{P}_{-i}) = \max_{p_i} \quad p_i \sum_{\mathbf{p}_{-i}} \sum_{\boldsymbol{\theta} \in \Theta} \mathbb{1}\Big(\theta_i - p_i \geq \max\{0, \max_{j \neq i}(\theta_j - p_j)\}\Big) \boldsymbol{P}_{-i}(\boldsymbol{\theta}, \mathbf{p}_{-i})$$

which is minimized when $\boldsymbol{P}_{-i}(\boldsymbol{\theta}, \mathbf{0}) = f(\boldsymbol{\theta})$ for all $\boldsymbol{\theta}$.

product to these consumers for free. The residual valuation of consumer $\boldsymbol{\theta} \in E_i$ for product i is $\theta_i - \max_{j \neq i} \theta_j$ and its distribution, denoted by $d_i : [0,1] \to [0,1]$, is:

$$d_i(\theta) := \sum_{\substack{\boldsymbol{\theta} \in E_i: \\ \theta_i - \max_{j \neq i} \theta_j = \theta}} f(\boldsymbol{\theta}) \quad \text{for all } \theta \in [0,1].$$

By charging a price p to consumers associated with message m_i, firm i faces demand $D_i(p) = \sum_{\theta \geq p} d_i(\theta)$ and obtains profit $\Pi_i(p) = pD_i(p)$. By selecting the optimal price $p_i^* = \operatorname{argmax}_p pD_i(p)$, firm i achieves exactly the lower bound profits $\underline{\Pi_i^*}$. We then have two possibilities.

The first possibility is that, for each firm i, the price p_i^* induces all consumers of types E_i to buy product i. In this case the outcome is efficient, and it is indeed optimal for other firms to charge a price of zero. Since producer-surplus is held down to its lower bound, this outcome is also consumer-optimal. When this is not the case, however, the outcome is inefficient. Nonetheless, we can modify the information structure to obtain an efficient outcome while, at the same time, holding down i's profits to $\underline{\Pi_i^*}$ and ensuring that other firms $j \neq i$ are still incentivized to charge a price of zero. This is done by partitioning the consumers with types $\theta \in E_i$ into submarkets to create a uniform profit preserving extremal segmentation under type distribution $d_i(\theta)$ as described by Bergemann, Brooks, and Morris (2015).

For a subset E_i^l of consumers with types in E_i, let d_i^l denote the distribution of residual valuation of consumers $\boldsymbol{\theta} \in E_i^l$ for product i and let $\operatorname{supp}(d_i^l)$ denote its support. Note that if E_i is partitioned in L groups $\{E_i^1, E_i^2, ..., E_i^L\}$ then the distribution of residual valuation d_i of E_i is such that $\sum_{l=1}^{L} d_i^l(\theta) = d_i(\theta)$ for all $\theta \in [0,1]$. We obtain

Theorem 2. The consumer-optimal information structure takes the following form:

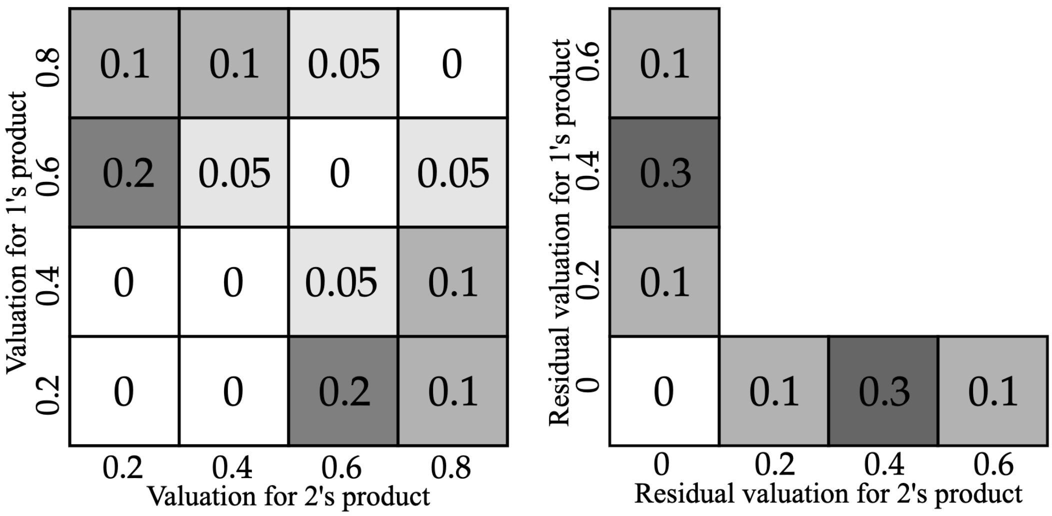
1. Consumers in Θ are partitioned into n groups $\{E_1, E_2, \ldots E_n\}$.
2. For each i, consumers in E_i are further partitioned into L_i groups $\{E_i^1, E_i^2, ..., E_i^{L_i}\}$ where $L_i < \infty$. Consumers in E_i^l are assigned the same message, and this message is distinct from that assigned to consumers in E_j^k for $i \neq j$ or $l \neq k$.
3. For each i and l,
$$\operatorname{supp}(d_i^l) = \operatorname*{argmax}_p p \sum_{\theta \geq p} d_i^l(\theta)$$
and
$$p_i^* \in \operatorname{supp}(d_i^l)$$
where p_i^* is an optimal price firm i selects facing a demand induced by residual valuations d_i of all consumers in E_i, i.e., $p_i^* \in \operatorname{argmax}_p p \sum_{\theta \geq p} d_i(\theta)$.

Theorem 2 shows that in the consumer-optimal information structure, firms $j \neq i$ are induced to set a price 0 to all consumers with types in E_i. Further, the E_i consumers are then partitioned into submarkets for firm i such that (i) for each type θ in a submarket, it is a profit maximizing choice for firm i to set a price for this submarket equal to $\theta_i - \max_{j \neq i} \theta_j$; (ii) one of the profit maximizing prices available to firm i is the same price firm i would set if it was forced to set a uniform price to all consumers with types in E_i; and (iii) in equilibrium, firm i charges the lowest price $\theta_i - \max_{j \neq i} \theta_j$ in the support of the submarket—hence all consumers in the submarket buys from i.

Our construction of the consumer-optimal information structure shares a similar economic logic to the construction of revenue-maximizing (bidder surplus-minimizing) information structure in Bergemann, Brooks, and Morris (2017) when bidders know their own value. In both cases the information designer publicly reveals the identity of the highest value player. This is the highest value bidder in the auction; in our setting, it is the firm that produces the consumer ideal product. By disclosing this information, the other players learn their comparative disadvantages which, in turn, intensify competition: in the auction the non-highest value bidders bid their value and in our setting the firms offering a non-ideal match to a consumer charge a price which is equal to their marginal cost.)

4.1 Consumer optimal outcome: example. Consider the distribution of consumer values shown in panel (a) of Figure 3 and, in panel (b), the distribution of residual valuations.

Figure 3: A distribution of consumers' types under which all firms publicly learn only which consumers prefer product i the most is not consumer-optimal.



(a) (b)

When firm 2 charges a price 0 to all consumers in E_1, there is a mass 0.1 of consumers in E_1 that are just willing to pay a price 0.6 for 1's product, a mass 0.3 that will pay a price 0.4 and a mass 0.1 that will pay a price of 0.2. Thus, firm 1's optimal price is $p_1^* = 0.4$. and this generates a profit of $\underline{\Pi_1^*} = 0.16$. However, this outcome is inefficient because it excludes the 0.1 mass of consumers in E_1 with residual valuation 0.2 (the 0.05 mass of consumers in E_1 with valuations $\theta_1 = 0.6$ and $\theta_2 = 0.4$, and the 0.05 mass of consumers with valuations $v_1 = 0.8$ and $\theta_2 = 0.6$).

Next consider the following alternative information structure. The consumers with types in E_1 are split in three groups and the designer sends a different message for each group.

Group E_1^1 contains all consumers with residual valuation 0.6 of product 1 and a mass 0.05 of consumers with residual valuation 0.4 for product 1. Group E_1^2 contains a 0.15 mass of consumers with residual valuation 0.4. Group E_1^3 contains the remaining 0.1 mass of consumers with residual valuation 0.4 and all consumers with residual valuation 0.2.

Suppose that, upon receiving the message for group E_1^l, firm 2 charges a price of 0. Given this it is easy to check that: (i) upon receiving the message for group E_1^1, firm 1 is indifferent between charging prices 0.4 and 0.6; (ii) upon receiving the message for group E_1^1, firm 1 charges a price 0; and (iii) upon receiving the message for group E_1^3, firm 1 is indifferent between charging prices 0.4 and 0.2. If firm 1 was to charge 0.4 to all such groups, the outcome would be the same as when all consumers with types in E_1 are put into the same group, and firm 1's profit would be equal to $\underline{\Pi_1^*} = 0.16$. However, if we instead always resolve firm 1's indifference in favor of charging the lower price, firm 1's profit is unaffected but the outcome is now efficient. Thus the upper bound on consumer surplus is obtained and we have a consumer-optimal information structure.

5 Efficient Information Structures

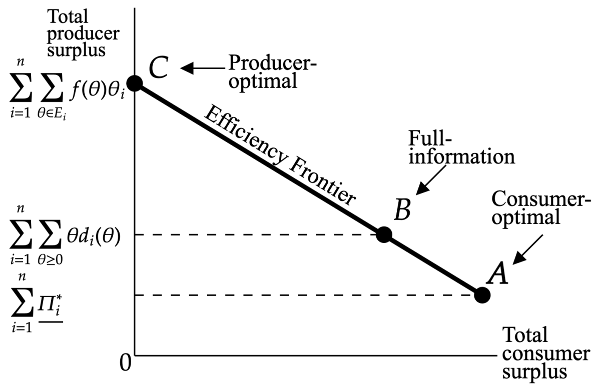
We showed that whenever the condition in Theorem 1 is met, the designer can allocate all available economic surplus to producers. We also showed that if the designer wishes to maximizes consumer surplus then, although it is not possible to allocate all available surplus to consumers, allocating as much as possible still leads to an efficient outcome. The consumer-optimal and producer-optimal outcomes are useful benchmarks, but it is also informative to consider what other points on the efficient frontier an information designer can obtain.

A first insight is that providing full information to all firms about all consumers (full information) always results in an efficient market outcome. Indeed, under full information there is an equilibrium in which each firm i sets a price 0 to all consumers in E_j for $j \neq i$, and charges each consumer in E_i her residual valuation. This is illustrated by point B in Figure 4. The consumer-optimal outcome, illustrated by point A in Figure 4, achieves a weakly higher proportion of consumer surplus. These are two points on the efficient frontier that can always be achieved. Hence, by assigning a fraction λ of consumers to the consumer-optimal information structure and a fraction $1-\lambda$ to the full information structure any outcome on the efficient frontier between points A and B can also be achieved. The next proposition gives tight conditions under which points A and B coincide exactly.

Proposition 3. The full information structure is consumer-optimal if and only if for all firms i, all consumers in E_i have the same residual valuation i.e., for each i and any pair $\boldsymbol{\theta}, \boldsymbol{\theta}' \in E_i$, $f(\boldsymbol{\theta}) > 0$, $f(\boldsymbol{\theta}') > 0$ implies $\theta_i - \max_{j\neq i} \theta_j = \theta_i' - \max_{j\neq i} \theta_j'$.

The proof of Proposition 3 is deferred to Appendix A.6. The condition requires that for all consumers in E_i, the differences between valuations for i and their second most favorite product must coincide. In the setting with 2 firms illustrated in Figure 1, the condition states that for E_1, only a single 'diagonal' e.g., types $\{(0.4, 0.2), (0.6, 0.4), (0.8, 0.6)\}$, can have positive masses of consumers, and likewise for E_2. This is a restrictive condition that

Figure 4: Efficient Information Structures



generically does not hold; we should therefore expect that the full information structure to be sub-optimal for consumers.

Finally, when the producer-optimal outcome (point C in Figure 4) is implementable (condition in Theorem 1 is met), then all points between point B and point C can also be obtained. To see this, suppose we wish to obtain a point $D = \lambda B + (1-\lambda)C$ for some $\lambda \in (0,1)$. We can partition the distribution of consumers f into $f_B(\boldsymbol{\theta}) = \lambda f(\boldsymbol{\theta})$ and $f_C(\boldsymbol{\theta}) = (1-\lambda)f(\boldsymbol{\theta})$ for each $\boldsymbol{\theta} \in \Theta$. We then apply the producer-optimal information structure to f_C (which has mass λ) and the full information structure to f_B (which has mass $1-\lambda$). Since the condition in Theorem 1 holds for f it also holds for f_C because this is simply a re-normalization of total mass.

The information structure we constructed to obtain point D allocates each type randomly between the full information structure and the producer-optimal information structure. This construction works when the condition in Theorem 1 holds. It remains an open question to find necessary and sufficient conditions for the existence of an information structure that obtains a given intermediate point between B and C.

6 Concluding Remarks

6.1 Takeaways for regulators. We have explored how platforms can use information on market participants to shape price competition. We provided conditions under which information grants the intermediary absolute power: by packaging information about consumers preferences in different ways, the platform can relax or intensify market competition to obtain any feasible ratio of consumer to producer surplus on the efficient frontier.

From the perspective of an antitrust authority mandated with protecting consumer surplus, this raises a delicate problem. Outrightly preventing the use of information will typically sacrifice efficiency while, without regulation on information disclosure, a platform that wishes to increase consumer surplus can intensify price competition well beyond the complete information case (hence achieving greater consumer surplus). At the same time, an intermediary with a revenue model based on monetizing consumer information may design an information structure that implements the same outcome perfect collusion would yield in an otherwise competitive downstream market.

The picture we have portrayed does not mean that regulators have no options. Our analysis shows that there are distinct principles on how information is disclosed which matter for attendant market outcomes. We therefore suggest that regulators might formulate guidelines or rules of conduct that ensure that such groups of consumers are formed in line with the principles characterizing the consumer-optimal information structure—i.e., only consumers with similar preferences (and hence the same most preferred product) should be grouped together and this information should be disclosed publicly to firms.

6.2 Price discrimination in practice. Our analysis is based on the assumption that firms will price discriminate if they can. If firms expect consumers to become aware of differential pricing based on consumers' willingness to pay, the ensuing reputational damage may deter the implementation of these practices. In this case, information design is irrelevant since firms must charge uniform prices.

There are, however, ways in which price discrimination can be concealed. First, a 2019 report by the UK's Digital Competition Experts Panel writes that if firms can "send secret deals to consumers, for example by directly offering discounts via email, the price discrimination becomes entirely opaque." The use of discount codes is widespread and encouraged by internet intermediaries.[17] In fact, when firms attempt to conceal price discrimination from consumers in this way it will be relatively challenging to detect it empirically. A web-scraping 'robot,' used in experiments like that run by Cavallo and Rigobon (2016) to compare online and offline prices, does not have the same web-surfing or purchase history as real profiles. As such, firms do not have the opportunity to target them with discount codes (for instance, through social media feeds). Second, in industries where the cost of providing the service being sold depends on the characteristics of the individual (e.g., insurance and credit markets), and in industries that use dynamic demand-based pricing (e.g., flights and ride-hailing), it is hard for consumers to understand what underlies price differences.[18] Again, in such cases, it is challenging for empirical work using publicly available data to identify price discrimination.

All this points to a lack of strong evidence for widespread price discrimination not necessarily implying that such practices are not taking place, albeit in more subtle ways. And this is why the possibility that consumer data are used to facilitate discriminatory pricing has drawn regulatory interest. China's new anti-monopoly guidelines—tailored exclusively to reigning in tech firms—explicitly outline the phenomena for data being used to "achieve coordinated behaviour" (State Administration for Market Regulation, 2021).[19] In a similar vein, a recent report by the Competition and Markets Authority in

[17]See Google's marketer playbook and Facebook's webpage for small businesses. Targeted discounts are also ubiquitous in the grocery market; supermarkets collect detailed data on consumers and price discriminates using coupons. Hannak et al. (2014) compare the prices charged to real consumer profiles obtained via Amazon Mechanical Turk. They find evidence that Home Depot, Sears, many travel sites (e.g. Cheaptickets, Orbitz, Priceline etc.) price discriminate.

[18]A 2018 report by the Competition and Markets Authority, the UK's competition regulator found that some home and motor insurance firms use complex and opaque pricing techniques to charge consumers with a higher willingness to pay markedly higher prices (Competition & Markets Authority, 2018).

[19]There is considerable anecdotal evidence for widespread price discrimination occurring in China. A survey conducted in 2019 by the Beijing Consumer Association finds that 88% of consumers believe that the practice of big data-enabled price discrimination is significant, and 57% have personally experienced this.

the UK reported that "even if there is limited evidence for personalized pricing, this could change quickly" (Competition & Markets Authority, 2021). Similar issues are highlighted in regulatory documents from the EU, US and Canada.[20]

6.3 Jointly designing information and access. In a companion paper (Elliott, Galeotti, Koh, and Li, 2022) we extend the present framework to allow the designer to jointly control the (possible many-to-many) matching between firms and consumers as well as the information firms receive about consumer preferences. Building upon the producer-optimal and consumer-optimal information structures, we give a complete characterization of all welfare outcomes achievable in equilibrium.

Our analysis also raises interesting questions for future work. While we consider a monopoly platform, in practice there are multiple internet companies, each with the ability to collect extensive proprietary data about consumers. This raises the prospect of competition among platforms. It would be interesting to study how this manifests, and what implications it has for the competitiveness of downstream markets. One possibility is that the internet companies compete for consumers and their information via their product offerings, while maintaining power over downstream market outcomes.

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Appendices

A Omitted Results and Proofs

A.1 Proof of Lemma 1.

Proof. Since $\mathcal{M} \supset \mathcal{M}'$, there is nothing to prove for the if direction. It remains to show that there exists a producer optimal information structure under message space $\mathcal{M}$, then there also exists a producer optimal information structure under message space $\mathcal{M}'$. For any $\boldsymbol{\psi} \in \boldsymbol{\Psi}^*$ and for each i, first observe that P requires

$$\text{supp}\big(\psi_i(\boldsymbol{\theta})\big) \cap \text{supp}\big(\psi_i(\boldsymbol{\theta}')\big) = \emptyset \quad \text{for all } \boldsymbol{\theta}, \boldsymbol{\theta}^{'} \in E_i \text{ such that } \theta_i \neq \theta_i^{'}.$$

Further, P requires that upon receiving message $m_i \in \text{supp}\big(\psi_i(\boldsymbol{\theta})\big)$ for $\boldsymbol{\theta} \in E_i$, firm i must find it optimal to charge price θ_i. As such, P also requires that

$$\bigcup_{\boldsymbol{\theta} \in \Theta} \text{supp}\big(\psi_i(\boldsymbol{\theta})\big) = \bigcup_{\boldsymbol{\theta} \in E_i} \text{supp}\big(\psi_i(\boldsymbol{\theta})\big).$$

If this were not true, then must exist $\boldsymbol{\theta}' \in E_j, j \neq i$ such that

$$\text{supp}\big(\psi_i(\boldsymbol{\theta}')\big) \setminus \bigcup_{\boldsymbol{\theta} \in E_i} \text{supp}\big(\psi_i(\boldsymbol{\theta})\big) \neq \emptyset$$

and firm i must receive messages in this set for some strictly positive mass of consumers. Upon receipt of this message, firm i can infer that the associated consumers are not in E_i. Property P requires that firm i only sell to consumers in E_i. But then firm i has a strictly profitable deviation by charging a price $0 < \varepsilon < v_1$, a contradiction.

Now for each firm i, and each type $\boldsymbol{\theta} \in E_i$, relabel all messages in $\text{supp}\big(\psi_i(\boldsymbol{\theta})\big)$ with the message $m_i = \theta_i$. By the above argument, all types $\boldsymbol{\theta} \in \Theta$ are then assigned some message in M_i', and this equilibrium continues to satisfy P. We can do this for each firm $i \in \mathcal{N}$ so that P is implemented with the messages $\mathcal{M}'$. □

A.2 Proof of Lemma 2. We implicitly imposed **Consistency** and **Separation** in the main text, arguing that it was without loss of generality to do so when the goal is to characterize the producer-optimal structure. Our proof will show that these two conditions, alongside **Firm-IC** and **Consumer-IC** are both sufficient and necessary. In particular, this will imply Lemma 2.

Proof. We first show that if the information structure satisfies **Consumer IC**, **Firm IC**, **Consistency** and **Separation** then it is producer-optimal. Suppose there exists an information structure that satisfies **Consumer IC**, **Firm IC**, **Consistency** and **Separation**. Consider a strategy profile in which all firms set prices equal to the messages they receive. Given **Separation**, $p_i = m_i = \theta_i$ for all $\boldsymbol{\theta} \in E_i$. Given Consistency and **Consumer IC** a consumer type $\boldsymbol{\theta} \in E_i$ buys from firm i because that consumer will never receive a price for product j that is less than her value θ_j. Hence, the outcome of this pricing strategy is producer-optimal. To see that the pricing strategy is an equilibrium note that **Consumer IC** guarantees that it is unprofitable for a firm i to deviate and set a price above m_i, and **Firm IC** guarantees it is unprofitable for a firm i to deviate and set a price below m_i.

We now show that if an information structure supports an equilibrium which is producer-optimal then there must exist an information structure that satisfies **Consumer IC**, **Firm IC**, **Consistency** and **Separation**.

By Lemma 1, if a producer-optimal information structure exists, we can find an information structure $\boldsymbol{\psi}^*$ which only sends messages in $\mathcal{M}'$ and induces an equilibrium satisfying P. Further, as in the equilibrium constructed in the proof of Lemma 1, all firms charge the prices equal to the message they receive. We now show that such an information structure necessarily satisfies the above conditions.

First, suppose $\boldsymbol{\psi}^*$ violates **Separation**. Then for some firm i, there exists $\boldsymbol{\theta} \in E_i$ such that $\psi_i^*(m_i|\boldsymbol{\theta}) > 0$ for some $m_i \in M_i', m_i \neq \theta_i$. But since in equilibrium, firms charge prices equal to the messages they receive, some strictly positive proportion of type $\boldsymbol{\theta}$ do not buy from i at price θ_i which violates P.

Second, $\boldsymbol{\psi}^*$ only sends messages in $\mathcal{M}'$, Hence **Consistency** must hold.

Third, suppose $\boldsymbol{\psi}^*$ violates **Consumer IC**. Then there exists firms i, j, type $\boldsymbol{\theta} \in E_j$, and message $m_i \in M_i'$ such that $\psi_i(m_i|\boldsymbol{\theta}) > 0$ and $\theta_i \geq m_i$ i.e., although type $\boldsymbol{\theta} \in E_j$ has valuation for i's product larger than m_i, some positive mass of them are nonetheless assigned the message m_i. If $\theta_i > m_i$, then since firms must charge prices equal to the messages they receive, it violates P. If $\theta_i = m_i$, then since $\boldsymbol{\theta} \in E_j$, P requires that such consumers buy from firm j at price θ_j. But then a positive mass of such types are exactly indifferent between firms i and j so no matter how ties are broken, at least one firm does not sell to all such consumers—this firm has a strictly profitable deviation to a lower price, violating P.

Finally, suppose $\boldsymbol{\psi}^*$ violates **Firm IC**. In equilibrium firms charge prices equal to the messages they receive, and all surplus is extracted from all consumers. Hence, a violation of **Firm IC** implies at least one firm has a strictly profitable deviation to a lower price. But this delivers strictly positive surplus to some consumers, violating P.

□

A.3 Proof of Theorem 1.

Proof. To show the "if" direction", we explicitly construct an information structure $\boldsymbol{\psi}$ which fulfils **Separation**, **Consumer IC**, **Firm IC**, and **Consistency**. By Lemma 2 this is sufficient.

We proceed by first constructing the marginals $(\psi_i)_i$, then defining the joint $\psi(\boldsymbol{\theta}) := \Pi_i \psi_i(\boldsymbol{\theta})$. If the consumer type $\boldsymbol{\theta} \in E_i$, firm i receives a message equal to the consumer's value for good i:

$$\psi_i(m_i|\boldsymbol{\theta}) := \begin{cases} 1 & \text{if } m_i = \theta_i \\ 0 & \text{otherwise.} \end{cases}$$

We note that this fulfils **Separation**.

We are left to specify messages sent to firm i when the consumer type is $\boldsymbol{\theta} \in \Theta \backslash E_i$. Recall that M_i' is the set of valuations for product i for consumers in set E_i. The construction of $\psi_i(m_i|\boldsymbol{\theta})$ for $\boldsymbol{\theta} \in \Theta \setminus E_i$ is analogous to the following matching problem:

- To satisfy **Consistency**, we want to match all consumers with types $\boldsymbol{\theta} \in \Theta \setminus E_i$ to values in the set M_i'.
- To satisfy **Consumer IC**, Each consumer type $\boldsymbol{\theta} \in \Theta \setminus E_i$ can only be matched to messages $m_i > \theta_i$.
- To satisfy **Firm IC**, the maximal mass of types belonging to $\Theta \setminus E_i$ with valuation for product i in interval $[\hat{p}_i, m_i)$ (for $\hat{p}_i < m_i$) that can be matched to a message $m_i \in M_i'$ is $G(\hat{p}_i, m_i)$, which we defined in main text as:

$$G(\hat{p}_i, m_i) = \frac{m_i - \hat{p}_i}{\hat{p}_i} \sum_{\boldsymbol{\theta}' \in E_i : \theta_i' = m_i} f(\boldsymbol{\theta}').$$

  We extend this definition so that $G(\hat{p}_i, m_i) = 0$ for $\hat{p}_i \geq m_i$.

Observe **Consumer IC** implies that for consumer type $\boldsymbol{\theta} \in \Theta \setminus E_i$ the higher is her value for product i the smaller is the set of messages she can be matched to. Hence, we construct the matching starting from types $\boldsymbol{\theta} \in \Theta \setminus E_i$ with the highest value for firm i's product.

Let the highest value for product i across types in $\Theta \setminus E_i$ be denoted by

$$v_{K_i} := \max\{\theta_i' | \boldsymbol{\theta}' \in \Theta \setminus E_i, f(\boldsymbol{\theta}') > 0\}.[21]$$

Take consumer $\boldsymbol{\theta} \in \Theta \setminus E_i$. Each message $m_i \leq \theta_i$ is always sent with probability zero. If $\theta_i = v_{K_i}$, each message $m_i > \theta_i$ is sent with probability:

$$\psi_i(m_i | \boldsymbol{\theta}) := \frac{G_i(v_{K_i}, m_i)}{H_i(v_{K_i})}.$$

Note that $\psi_i(m_i|\boldsymbol{\theta})$ depends on $\boldsymbol{\theta}$ only through θ_i; we abuse notation and use $\psi_i(m_i|\theta_i)$ to represent $\psi_i(m_i|\boldsymbol{\theta})$. If $K_i = 1$, our construction of $\psi_i(m_i|\boldsymbol{\theta})$ is complete, recalling that we defined

$$H_i(v) = \sum_{m_i > v} G_i(v, m_i)$$

in the main text.

Now suppose $K_i > 1$. If $\theta_i = v_{K_i - 1}$, each message $m_i > \theta_i$ is going to be sent with probability:

$$\psi_i(m_i | \boldsymbol{\theta}) := \frac{G_i(v_{K_i - 1}, m_i) - \psi_i(m_i | v_{K_i}) Q_i(v_{K_i})}{H_i(v_{K_i - 1}) - Q_i(v_{K_i})},$$

where $Q_i(c)$ denotes the mass of consumers in $\Theta \setminus E_i$ with a value for product i above cutoff c, i.e.,

$$Q_i(c) := \sum_{\substack{\boldsymbol{\theta}' \in \Theta \setminus E_i : \\ \theta_i' \geq c}} f(\boldsymbol{\theta}').$$

[21] This is well defined: because the condition in Theorem 1 is satisfied, there is positive mass on E_i for each firm i hence the set we take the maximum over is non-empty.

If $K_i = 2$, our construction of $\psi_i(m_i|\boldsymbol{\theta})$ is completed.

Suppose $K_i > 2$; The construction proceeds iteratively: for each $2 \leq k \leq K_i$, after we have constructed $\psi_i(m_i|v_{K_i-t})$ for $t = 0 \dots k-1$, when $\theta_i = v_{K_i-k}$, each message $m_i > \theta_i$ is sent with probability:

$$\psi_i(m_i|\boldsymbol{\theta}) := \frac{G_i(v_{K_i-k}, m_i) - \sum_{t=0}^{t=k-1} \psi_i(m_i|v_{K_i-t})[Q_i(v_{K_i-t}) - Q_i(v_{K_i-t+1})]}{H_i(v_{K_i-k}) - Q_i(v_{K_i-k+1})}.$$

Notice that $Q_i(v_{K_i}) = Q_i(v_{K_i}) - Q_i(v_{K_i+1})$, since by definition, $v_{K_i} < v_K$ and $Q_i(v_{K_i+1}) = 0$. Hence, $\psi_i(m_i|v_{K_i-1})$ can also be represented in the above form.

Intuitively, the denominator is difference between the total capacity than can be used to match consumer types in $\Theta \setminus E_i$ with value v_{K_i-k} and the capacity already used to match consumers types in $\Theta \setminus E_i$ with a higher value for product i. The numerator is the analogous object for local capacity i.e., those associated with message m_i. Indeed, $\psi_i(m_i|v_{K_i-t})[Q_i(v_{K_i-t}) - Q_i(v_{K_i-t+1})]$ is the local capacity already used to match consumer types with value v_{K_i-t} for product i. Hence the probability that m_i is sent is the ratio of the leftover local capacity to the total leftover capacity.

The construction of $\psi_i(m_i|\boldsymbol{\theta})$ already takes care of **Separation**, **Consistency** and **Consumer IC**. Hence, we only need to verify $\psi_i(m_i|\boldsymbol{\theta})$ is a valid probability and it satisfies **Firm IC**.

Firm IC requires for each $m_i \in M'_i$ and each $\hat{p}_i < m_i$,

$$G(\hat{p}_i, m_i) \geq \sum_{\boldsymbol{\theta}' \in \Theta \setminus E_i : \theta'_i \geq \hat{p}_i} \psi_i(m_i|\boldsymbol{\theta}') f(\boldsymbol{\theta}').$$

It is sufficient the check that the above holds for $\hat{p}_i \in V : \hat{p}_i \leq v_{K_i}$ since deviating to a price not in V under the conjecture that all other firms are obeying their pricing recommendations is dominated. For each $v_k < m_i$ and $k \leq K_i$,

$$\begin{aligned}
\sum_{\boldsymbol{\theta}' \in \Theta \setminus E_i : \theta'_i \geq v_k} \psi_i(m_i|\boldsymbol{\theta}') f(\boldsymbol{\theta}') &= \sum_{t=k}^{K_i} \psi_i(m_i|v_t)[Q_i(v_t) - Q_i(v_{t+1})] \\
&= \psi_i(m_i|v_k)[Q_i(v_k) - Q_i(v_{k+1})] \\
&\quad + \sum_{t=k+1}^{K_i} \psi_i(m_i|v_t)[Q_i(v_t) - Q_i(v_{t+1})]^{22} \\
&\leq G(v_k, m_i) - \sum_{t=0}^{K_i-k-1} \psi_i(m_i|v_{K_i-t})[Q_i(v_{K_i-t}) - Q_i(v_{K_i-t+1})] \\
&\quad + \sum_{t=k+1}^{K_i} \psi_i(m_i|v_t)[Q_i(v_t) - Q_i(v_{t+1})] \\
&= G(v_k, m_i)
\end{aligned}$$

[22] Define the sum to be 0 if $k = K_i$.

Note that the first inequality follows because

$$\psi_i(m_i|v_k)[Q_i(v_k) - Q_i(v_{k+1})] = \left(\frac{G_i(v_k, m_i) - \sum_{t=0}^{t=K_i-k-1} \psi_i(m_i|v_{K_i-t})[Q_i(v_{K_i-t}) - Q_i(v_{K_i-t+1})]}{H_i(v_k) - Q_i(v_{k+1})} \right) \times [Q_i(v_k) - Q_i(v_{k+1})]^{23}$$

$$\leq G_i(v_k, m_i) - \sum_{t=0}^{t=K_i-k-1} \psi_i(m_i|v_{K_i-t})[Q_i(v_{K_i-t}) - Q_i(v_{K_i-t+1})]$$

where the inequality holds because the assumption that condition in Theorem 1 is satisfied implies $H_i(v_k) - Q_i(v_{k+1}) \geq Q_i(v_k) - Q_i(v_{k+1})$. Hence, **Firm IC** is satisfied.

Finally, we show that $\psi_i(m_i|\boldsymbol{\theta})$ is a valid probability mass function. From the above we have that for all $k \leq K_i$,

$$\sum_{t=k}^{t=K_i} \psi_i(m_i|v_t)[Q_i(v_t) - Q_i(v_{t+1})] \leq G(v_k, m_i).$$

Hence,

$$\sum_{t=0}^{t=k-1} \psi_i(m_i|v_{K_i-t})[Q_i(v_{K_i-t}) - Q_i(v_{K_i-t+1})] \leq G_i(v_{K_i-k+1}, m_i),$$

which implies that the numerator of $\psi_i(m_i|v_{K_i-k})$ is non-negative. To see its denominator is strictly positive note that:

$$H_i(v_{K_i-k}) > H_i(v_{K_i-k+1}) \geq Q_i(v_{K_i-k+1}),$$

where the last inequality follows because, by assumption, the condition in Theorem 1 is satisfied. It remains to show that $\psi_i(m_i|v_{K_i-k})$ sums to one for all k. Observe

$$\sum_{m_i \in M_i'} \psi_i(m_i|v_{K_i-k}) = \sum_{m_i \in M_i'} \left(\frac{G_i(v_{K_i-k}, m_i) - \sum_{t=0}^{t=k-1} \psi_i(m_i|v_{K_i-t})[Q_i(v_{K_i-t}) - Q_i(v_{K_i-t+1})]}{H_i(v_{K_i-k}) - Q_i(v_{K_i-k+1})} \right)$$

$$= \frac{H_i(v_{K_i-k}) - \sum_{t=0}^{t=k-1} \sum_{m_i \in M_i'} \psi_i(m_i|v_{K_i-t})[Q_i(v_{K_i-t}) - Q_i(v_{K_i-t+1})]}{H_i(v_{K_i-k}) - Q_i(v_{K_i-k+1})}.$$

We show this by inducting on k. For $k = 0$, $\sum_{m_i \in M_i'} \psi_i(m_i|v_{K_i}) = 1$ straightforwardly. Now suppose that this holds for $k - 1$, then from the telescoping sum, it also holds for k:

$$\sum_{m_i \in M_i'} \psi_i(m_i|v_{K_i-k}) = 1.$$

We now turn to the "only if" direction. Towards a contradiction suppose the condition is violated but $\boldsymbol{\Psi}^* \neq \emptyset$. By Lemma 2 there exists an information structure $\boldsymbol{\psi} \in \boldsymbol{\Psi}^*$ satisfying **Separation** with corresponding message functions $(\psi_i)_i$, satisfying **Consistency**, **Consumer IC** and **Firm IC**.

[23] Again, define the sum to be zero if $k = K_i$.

Given any such message functions $(\psi_i)_i$, consider the distribution function it generates, $(\tilde{G}_i)_i$ which fulfils

$$\tilde{G}_i(\hat{\theta}_i, m_i) = \sum_{\substack{\boldsymbol{\theta}\in\Theta\setminus E_i: \\ \theta_i \geq \hat{\theta}_i}} \psi_i(m_i|\boldsymbol{\theta}) f(\boldsymbol{\theta}) \quad \text{for all } m_i \in M_i',\ \hat{\theta}_i \in V.$$

As the condition in Theorem 1 does not hold, there must exist a firm i and alternative price $\tilde{\theta}_i$ that i can set such that

$$\sum_{\substack{\boldsymbol{\theta}'\in\Theta\setminus E_i: \\ \theta_i' \geq \tilde{\theta}_i}} f(\boldsymbol{\theta}') > H_i(\tilde{\theta}_i) = \sum_{m_i\in M_i'} G_i(\tilde{\theta}_i, m_i) \geq \sum_{m_i\in M_i'} \tilde{G}_i(\tilde{\theta}_i, m_i),$$

where the strict inequality follows from the fact that the condition in Theorem 1 is not fulfilled and the weak inequality follows from **Firm IC**. But by **Consistency**,

$$\sum_{\substack{\boldsymbol{\theta}'\in\Theta\setminus E_i: \\ \theta_i' \geq \tilde{\theta}_i}} f(\boldsymbol{\theta}') = \sum_{m_i\in M_i'} \tilde{G}_i(\tilde{\theta}_i, m_i),$$

a contradiction.

□

A.4 Proof of Proposition 1.

Proof. Let H and $\tilde{H}$ be the corresponding functions defined in the main text for distribution f and $\tilde{f}$, respectively. Suppose $\boldsymbol{\Psi}^* \neq \emptyset$ under the distribution f and fix $\hat{\theta}_i \in (v_{k-1}, v_k]$ for some $1 \leq k \leq K$, where we set $v_0 = 0$ and $v_{K+1} = 1$. We have

$$\begin{aligned}
H_i(\hat{\theta}_i) &= \sum_{m_i\in M_i'} G_i(\hat{\theta}_i, m_i) = \sum_{\substack{m_i\in M_i': \\ m_i\geq\hat{\theta}_i}} \left(\frac{m_i - \hat{\theta}_i}{\hat{\theta}_i} \sum_{\substack{\boldsymbol{\theta}'\in E_i: \\ \theta_i' = m_i}} f(\boldsymbol{\theta}') \right) \\
&= \sum_{K\geq l\geq k} \left(\frac{v_l - \hat{\theta}_i}{\hat{\theta}_i} \sum_{\substack{\boldsymbol{\theta}'\in E_i: \\ \theta_i' = v_l}} f(\boldsymbol{\theta}') \right) = \sum_{K\geq l\geq k} \left(\frac{v_l - \hat{\theta}_i}{\hat{\theta}_i} \right) \Bigg(\underbrace{\sum_{\substack{\boldsymbol{\theta}'\in E_i: \\ \theta_i' \geq v_l}} f(\boldsymbol{\theta}')}_{:= a_l} - \underbrace{\sum_{\substack{\boldsymbol{\theta}'\in E_i: \\ \theta_i' \geq v_{l+1}}} f(\boldsymbol{\theta}')}_{:= a_{l+1}} \Bigg) \\
&= \frac{1}{\hat{\theta}_i} \sum_{K\geq l\geq k} v_l (a_l - a_{l+1}) - \sum_{K\geq l\geq k} (a_l - a_{l+1}) = \left(\frac{v_k}{\hat{\theta}_i} - 1 \right) a_k + \frac{1}{\hat{\theta}_i} \sum_{K\geq l\geq k} (v_{l+1} - v_l) a_{l+1} \\
&\leq \left(\frac{v_k}{\hat{\theta}_i} - 1 \right) b_k + \frac{1}{\hat{\theta}_i} \sum_{K\geq l\geq k} (v_{l+1} - v_l) b_{l+1} = \tilde{H}_i(\hat{\theta}_i)
\end{aligned}$$

where here

$$b_k := \sum_{\substack{\boldsymbol{\theta}'\in E_i: \\ \theta_i' \geq v_k}} \tilde{f}(\boldsymbol{\theta}') \geq \sum_{\substack{\boldsymbol{\theta}'\in E_i: \\ \theta_i' \geq v_k}} f(\boldsymbol{\theta}') := a_k$$

for all $1 \leq k \leq K+1$ by condition (i) of Proposition 1. The inequality follows because (i) $v_k/\hat{\theta}_i - 1 \geq 0$; (ii) $v_{l+1} - v_l > 0$; and (iii) $b_k \geq a_k$. The last equality follows from the same expansion of H_i, but replacing a with b.

Furthermore, condition (ii) of Proposition 1 implies that

$$\sum_{\substack{\boldsymbol{\theta}' \in \Theta \setminus E_i: \\ \theta'_i \geq \hat{\theta}_i}} f(\boldsymbol{\theta}') = \sum_{j \neq i} \sum_{\substack{\boldsymbol{\theta}' \in E_j: \\ \theta'_i \geq \hat{\theta}_i}} f(\boldsymbol{\theta}') \geq \sum_{j \neq i} \sum_{\substack{\boldsymbol{\theta}' \in E_j: \\ \theta'_i \geq \hat{\theta}_i}} \tilde{f}(\boldsymbol{\theta}') = \sum_{\substack{\boldsymbol{\theta}' \in \Theta \setminus E_i: \\ \theta'_i \geq \hat{\theta}_i}} \tilde{f}(\boldsymbol{\theta}').$$

Hence,

$$\tilde{H}_i(\hat{\theta}_i) \geq \sum_{\substack{\boldsymbol{\theta}' \in \Theta \setminus E_i: \\ \theta'_i \geq \hat{\theta}_i}} \tilde{f}(\boldsymbol{\theta}') \quad \text{for all } \hat{\theta}_i \in (0, v_K]$$

and so, by Theorem 1, $\boldsymbol{\Psi^*} \neq \emptyset$ under $\tilde{f}$. □

A.5 Proof of Proposition 2.

Proof of Proposition 2. Let $\bar{v}^n$ be the highest value in the support of f i.e., $\bar{v}^n = \max \operatorname{supp} f$ where we use the coordinate-wise maximum. Fix an arbitrary information structure $\boldsymbol{\psi}$ and first observe that in any equilibrium induced by $\boldsymbol{\psi}$, the consumer must buy from some firm with strictly positive probability otherwise all firms make zero profits and it is strictly profitable for any firm to deviate uniformly to the price $\bar{v} - \epsilon$ for some $\epsilon > 0$.

Next define the following equilibrium object

$$F^{\boldsymbol{\psi}}(x) := \mathbb{P}(\text{The consumer pays price } p \leq x | \text{the consumer buys one of the products}).$$

We will now show that the highest price in the support of $F^{\boldsymbol{\psi}}$, $\bar{p} := \inf\{p \in [0,1] : F^{\boldsymbol{\psi}}(p) = 1\}$ must be zero. To this end, suppose, towards a contradiction, that $\bar{p} > 0$. Now define

$$F_i^{\boldsymbol{\psi}} := \mathbb{P}\left(\begin{matrix}\text{The consumer pays price } p \leq x \\ \text{and buys from } i\end{matrix} \middle| \text{the consumer buys one of the products}\right),$$

noting that for any $p \in [0,1]$, $F^{\boldsymbol{\psi}}(p) = \sum_{i=1}^{n} F_i^{\boldsymbol{\psi}}(p)$. Since $\bar{p}$ was defined as the highest point in the support of $F^{\boldsymbol{\psi}}$, choose $\epsilon > 0$ sufficiently small such that

$$F^{\psi}(\bar{p}) - F^{\psi}(\bar{p} - \epsilon) = \sum_{i=1}^{i=n} [F_i^{\psi}(\bar{p}) - F_i^{\psi}(\bar{p} - \epsilon)] > 0.$$

There must then exist some firm i such that

$$F_i^{\psi}(\bar{p}) - F_i^{\psi}(\bar{p} - \epsilon) \leq \frac{1}{n}[F^{\psi}(\bar{p}) - F^{\psi}(\bar{p} - \epsilon)].$$

Now consider the following uniform downward deviation for i:[24]whenever it would have chosen price $p \in (\bar{p} - \epsilon, 1]$, charge price $\bar{p} - \epsilon$ instead; if it would have chosen price $p \leq \bar{p} - \epsilon$, leaves prices unchanged. We conclude by showing that this deviation is strictly profitable.

[24]Feldman, Lucier, and Nisan (2016); Bergemann, Brooks, and Morris (2017) also consider uniform upward deviations in auction settings.

Note that i's loss on the extensive margin—the reduction in price charged to consumers she previously sold to—is at most

$$\epsilon[F_i^{\psi,E}(\bar{p}) - F_i^{\psi,E}(\bar{p}-\epsilon)];$$

on the other hand, the business stealing gain is at least

$$(\bar{p}-\epsilon)\sum_{j\neq i}[F_j^{\psi,E}(\bar{p}) - F_j^{\psi,E}(\bar{p}-\epsilon)]$$

since by deviating to $\bar{p}-\epsilon$, firm i now poaches all consumers who were previously buying from some firm $j \neq i$ at prices strictly greater than $\bar{p}-\epsilon$. For this to be an equilibrium induced by $\boldsymbol{\psi}$, a necessary condition is that for all uniform downward deviations to $\bar{p}-\epsilon$ for $\epsilon > 0$,

$$\epsilon[F_i^{\psi,E}(\bar{p}) - F_i^{\psi,E}(\bar{p}-\epsilon)] \geq (\bar{p}-\epsilon)\sum_{j\neq i}[F_j^{\psi,E}(\bar{p}) - F_j^{\psi,E}(\bar{p}-\epsilon)].$$

But this implies

$$\frac{\bar{p}-\epsilon}{\epsilon} \leq \frac{F_i^{\psi,E}(\bar{p}) - F_i^{\psi,E}(\bar{p}-\epsilon)}{\sum_{j\neq i}[F_j^{\psi,E}(\bar{p}) - F_j^{\psi,E}(\bar{p}-\epsilon)]} \leq \frac{1}{n-1}$$

which is a contradiction for sufficiently small $\epsilon > 0$. □

A.6 Proof of Proposition 3.

Proof. If: If the condition in Proposition 3 holds, then this implies that for each firm i, d_i has singleton support, so full information is both efficient and yields profits $\underline{\Pi}^*$ for each firm which implies that it is consumer-optimal.

Only if: Suppose there exists some pair $\boldsymbol{\theta}, \boldsymbol{\theta}' \in E_i$ such that

$$a = \theta_i - \max_{j\neq i}\theta_j > \theta'_i - \max_{j\neq i}\theta'_j = b.$$

This implies that the support of d_i includes at least a and b. It will suffice to restrict our attention to these two residual valuations.

Under the full information design, firm i makes $ad_i(a) + bd_i(b)$ from these two points. Now consider a modification of the full information structure which continues to give full information about types with residual valuation not in $\{a, b\}$. For residual valuations a, b, we now group all consumers with residual valuations equal to b, as well as mass $\varepsilon > 0$ of consumers with residual valuations equal to a together. Firm i continues to find it optimal to set a price equal to b for this group since

$$b(d_i(b) + \varepsilon) \geq a\varepsilon$$

for sufficiently small ε. We group the remaining mass of consumers with residual valuations equal to a in a separate group. Now firm i makes profits

$$b(d_i(b) + \varepsilon) + a(d_i(a) - \varepsilon)$$

from the types in E_i with residual valuations a, b, and profits from all other types remain unchanged. As such, it makes $\varepsilon(a - b) > 0$ less than under the full information design. But since this equilibrium is efficient, total consumer surplus is strictly higher than under full information. □

B Online Appendix to 'Market Segmentation through Information'

Matthew Elliott, Andrea Galeotti, Andrew Koh, and Wenhao Li

B.1 Ability to restrict access to consumers aids producer-optimal outcomes. Suppose that, in addition to designing information, the information designer can restrict the set of consumers that each firm can make price offers to. This might correspond to the case in which the intermediary provides exclusive opportunities for downsteam firms to reach certain consumers. This contrasts with our setting in which downstream firms have the means to independently make price offers to all consumers. As Bergemann and Bonatti (2019) emphasise, this distinction is pivotal for understanding market outcomes. In this appendix, we develop a simple generalization of the model in which the intermediary can restrict certain firms from accessing certain customers. We show that this weakens the conditions under which a producer-optimal information structure exists.

Let $R_i : \Theta \to [0,1]$, $R_i \leq f$ denote the distribution of the mass of consumers' types that the designer can prevent firm i from accessing. We view this as a primitive of the model. In essence, the information designer can decide not to send messages to firm i for certain consumer types and, in that case, firm i cannot sell to these consumers. The model developed in the main text corresponds to the case when $R_i(\boldsymbol{\theta}) = 0$ for all $\boldsymbol{\theta}$ and for all firms i. Fixing $(f, (R_i)_{i=1}^n)$, we wish to study when the producer-optimal outcome is achievable.

A first observation is that to induce a producer-optimal outcome, the designer must always allow firm i to make price offers to consumer types in E_i. Hence, we restrict attention to cases in which for all $i \in \mathcal{N}$, $R_i(\boldsymbol{\theta}) = 0$ for all $\boldsymbol{\theta} \in E_i$. A second observation is that since the designer needs to prevent firm i from undercutting firms $j \neq i$, it is without loss to assume that firm i faces the distribution $f(\boldsymbol{\theta}) - R_i(\boldsymbol{\theta})$ for all $\boldsymbol{\theta} \notin E_i$.

Proposition 4. If $\boldsymbol{\Psi}^* \neq \emptyset$ under $(f, (R_i)_{i=1}^n)$, then for any $(\tilde{R}_i)_{i=1}^n$ such that

$$\sum_{\substack{\boldsymbol{\theta}' \in \Theta \setminus E_i: \\ \theta'_i \geq \hat{\theta}_i}} R_i(\boldsymbol{\theta}') \leq \sum_{\substack{\boldsymbol{\theta}' \in \Theta \setminus E_i: \\ \theta'_i \geq \hat{\theta}_i}} \tilde{R}_i(\boldsymbol{\theta}') \qquad \text{for all } \hat{\theta}_i \in V, \text{ and all } i \in \{1, 2, \ldots, n\},$$

$\boldsymbol{\Psi}^* \neq \emptyset$ under $(f, (\tilde{R}_i)_{i=1}^n)$.

Proof. Fix a firm $i \in \mathcal{N}$. By our second observation above, the relevant distribution of consumers over $\Theta \setminus E_i$ to consider is $(f - R_i)$, the remaining mass of consumers the designer cannot prevent firm i from accessing.

We obtain that for all $i \in \mathcal{N}$ and for all $\hat{\theta}_i \in V$

$$H_i(\hat{\theta}_i) \geq \sum_{\substack{\boldsymbol{\theta}' \in \Theta \setminus E_i: \\ \theta'_i \geq \hat{\theta}_i}} \Big(f_i(\boldsymbol{\theta}') - R(\boldsymbol{\theta}') \Big) \geq \sum_{\substack{\boldsymbol{\theta}' \in \Theta \setminus E_i: \\ \theta'_i \geq \hat{\theta}_i}} \Big(f_i(\boldsymbol{\theta}') - \tilde{R}(\boldsymbol{\theta}') \Big)$$

where the first inequality follows from Theorem 1 because $\boldsymbol{\Psi}^* \neq \emptyset$ and the second inequality follows because of the condition in the Proposition 4. Hence, $\boldsymbol{\Psi}^* \neq \emptyset$ under $(f, (\tilde{R}_i)_{i=1}^n)$. □

Clearly, whenever $R_i \leq \tilde{R}_i$ pointwise, the condition in Proposition 4 is satisfied. This implies that the ability to restrict access to consumers is complementary to achieving producer-optimal outcomes through information design. Further, whenever

$$\sum_{\boldsymbol{\theta}' \in \Theta \setminus E_i} R_i(\boldsymbol{\theta}') = \sum_{\boldsymbol{\theta}' \in \Theta \setminus E_i} \tilde{R}_i(\boldsymbol{\theta}'),$$

the relation in Proposition 4 reduces to one of first-order stochastic dominance over $\Theta \setminus E_i$. This implies that to achieve the producer-optimal outcomes, it is more valuable for the designer to be able to restrict i's access to consumers who, while not having i as their most preferred product, nonetheless value i's product highly.

B.2 Continuous version of the model. In this Online Appendix, we develop a continuous version of our model, and state and prove analogues of Theorems 1 and 2.

As before, there is a finite set of firms, indexed $\mathcal{N} = \{1, \ldots, n\}$ each of which produces a single product at zero cost. There is once again a continuum of consumers with unit mass, each of whom demands a single unit inelastically. Valuations are now the interval $V = [0, 1]$. A consumer of type $\boldsymbol{\theta} := (\theta_1, \ldots, \theta_n) \in V^n$ obtains utility $\theta_i \in V$ purchasing from firm i. Our type space is then $V^n := [0, 1]^n$ and the distribution of consumers over V^n is distributed according to $f : V^n \to \mathbb{R}_{>0}$ with bounded density and full support.

As before, we will develop notation to describe the types for which it is efficient to buy from firm i. To this end, define n subsets of V^n, $\{E_1, E_2, \ldots, E_n\}$ where $E_i := \{\boldsymbol{\theta} \in V^n : \theta_i > \theta_j, j \neq i\}$. Note that $V^n \setminus \bigcup_{i=1}^n E_i$ is of zero measure.

The information designer now chooses a message function

$$\boldsymbol{\psi} : V^n \to \Delta(\mathcal{M})$$

where, $\mathcal{M} := \prod_{i=1}^n M_i$ and $M_i := [0, 1]$ is the message space for firm i, and $\Delta(\mathcal{M})$ is the set of feasible probability distributions over the joint message space. A strategy for firm i is $\sigma_i : M_i \to \Delta([0, 1])$. Let $\psi_i(\boldsymbol{\theta})$ denote the marginal distribution of $\boldsymbol{\psi(\theta)}$ over the set of messages received by firm i. Let $x_i(\cdot|\boldsymbol{\theta}) : \mathcal{B}([0, 1]) \to [0, 1]$ denote the Borel measure assigning proportion $x_i(B|\boldsymbol{\theta})$ of messages to the Borel set $B \in \mathcal{B}([0, 1])$ given type $\boldsymbol{\theta}$. We let $\psi_i(m_i|\boldsymbol{\theta})$ denote the density of messages given the type $\boldsymbol{\theta} \notin E_i$ whenever this density exists i.e.,

$$\int_{m_i \in B} \psi_i(m_i|\boldsymbol{\theta}) dm_i = x_i(B|\boldsymbol{\theta}) \quad \text{for all } B \in \mathcal{B}([0, 1]).$$

B.2.1 Producer-optimal design in the continuous case.

We once again wish to characterize the set of distributions under which there exists some information structure which can induce an equilibrium under which all available suplus from trade is extracted as producer surplus. However, we modify condition P in the main text to disregard zero-measure sets of consumers:

P^C **(Full Surplus Extraction)** Consumers of almost all types $\boldsymbol{\theta} \in V^n$ pay $\max_{i \in \mathcal{N}} \theta_i$.

As in the main text, we will let $\mathbf{\Psi}^*$ denote the set of information structures which induce an equilibrium fulfilling P^C. We now extend several properties of information structures to the continuous case. For all $i \in \mathcal{N}$,

$$x_i(\{\theta_i\}|\boldsymbol{\theta}) = 1 \quad \text{for almost all } \boldsymbol{\theta} \in E_i. \tag{Separation}$$

This says that except for zero measure subsets of E_i, firm i receives a precise message θ_i for the type $\boldsymbol{\theta} \in E_i$.

$$\int_{\boldsymbol{\theta} \notin E_i} x_i([0, \theta_i]|\boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta} = 0, \quad \text{for all } i \in \mathcal{N}. \tag{Consumer IC}$$

This states that almost all consumers outside of E_i are assigned messages greater than their valuation for i's product.

For all $i \in \mathcal{N}$, and $\boldsymbol{\theta} \notin E_i$, choose $x_i(\cdot|\boldsymbol{\theta})$ so that the density $\psi_i(m_i|\boldsymbol{\theta})$ exists, and for almost all $m_i \in (0,1)$, and all $\hat{p}_i < m_i$,

$$\underbrace{(m_i - \hat{p}_i) \int_{\boldsymbol{\theta'} \in E_i : \theta'_i = m_i} f(\boldsymbol{\theta'}) d^{n-1} \boldsymbol{\theta'}}_{\text{Inframarginal losses}} \geq \underbrace{\hat{p}_i \int_{\boldsymbol{\theta} \in V^n \setminus E_i : \theta_i \geq \hat{p}_i} \psi_i(m_i|\boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta}}_{\text{Business stealing gains}} \tag{Firm IC - sufficient}$$

which states that for each realization of (almost all) types $\boldsymbol{\theta} \in E_i$, firm i must weakly prefer to follow the recommendation to charge price $m_i = \theta_i$ over deviating to any lower price $\hat{p}_i < m_i$.

We aggregate **Firm IC - sufficient** over all messages in the set $(\hat{p}_i, 1]$ by requiring that for all $i \in \mathcal{N}$ and all $\hat{p}_i \in (0,1)$

$$\int_{m_i > \hat{p}_i} \left((m_i - \hat{p}_i) \int_{\boldsymbol{\theta'} \in E_i : \theta'_i = m_i} f(\boldsymbol{\theta'}) d^{n-1} \boldsymbol{\theta'} \right) dm_i \geq \hat{p}_i \int_{\boldsymbol{\theta} \notin E_i : \theta_i \geq \hat{p}_i} x_i((\hat{p}_i, 1]|\boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta}. \tag{Firm IC-necessary}$$

This condition is necessary for an information structure to be producer-optimal, because if it were not fulfilled, then upon receipt of messages in $(\hat{p}_i, 1]$, firm i strictly prefers to charge some price $\hat{p}_i$, and captures extra mass of consumers equal to the integral on the right hand side (which is strictly positive), violating P^C.

Lemma $2^{\mathbf{C}}$. If there exists an information design fulfills **Consumer IC**, **Separation** and **Firm IC - sufficient**, then $\mathbf{\Psi}^* \neq \emptyset$; if $\mathbf{\Psi}^* \neq \emptyset$, then there exists an information design fulfilling **Consumer IC**, **Separation** and **Firm IC - necessary**.

As in the discrete case, it is helpful to develop notation to describe the maximum mass of consumers over $V^n \setminus E_i$ which can be matched to a given message $m_i > 0$ for firm i in an incentive-compatible manner.

$$G_i(\hat{p}_i, m_i) := \begin{cases} \dfrac{m_i - \hat{p}_i}{\hat{p}_i} \displaystyle\int_{\boldsymbol{\theta'} \in E_i : \theta'_i = m_i} f(\boldsymbol{\theta'}) d^{n-1} \boldsymbol{\theta'} & \text{if } \hat{p}_i \leq m_i \\ 0 & \text{otherwise} \end{cases}$$

As before, for $\hat{p}_i \in (0,1)$, define

$$H_i(\hat{p}_i) := \int_{m_i \in M_i} G_i(\hat{p}_i, m_i) dm_i = \int_{m_i > \hat{p}_i} G_i(\hat{p}_i, m_i) dm_i.$$

Function H_i gives maximum mass of consumers over $V^n \setminus E_i$ with valuation for good i above cutoff $\hat{p}_i$ which can ever be matched to types in E_i in an incentive-compatible manner.

Theorem $1^{\mathbf{C}}$. $\mathbf{\Psi}^* \neq \emptyset$ if and only if for all $i \in \mathcal{N}$ and all $\hat{p}_i \in V$,

$$H_i(\hat{p}_i) \geq \int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \theta'_i \geq \hat{p}_i} f(\boldsymbol{\theta}') d^n \boldsymbol{\theta}'$$

Proof. We begin by showing that if the condition is fulfilled, then $\mathbf{\Psi}^* \neq \emptyset$. For each $i \in \mathcal{N}$ and $\boldsymbol{\theta} \in V^n$, we directly construct $\psi_i(\boldsymbol{\theta})$ then define $\psi(\boldsymbol{\theta}) := \Pi_i \psi_i(\boldsymbol{\theta})$.

Notice that the construction of $\psi_i(\boldsymbol{\theta})$ in the discrete case proceeded iteratively; this does not work in the continuous case. Instead, we adopt an alternate approach. Define:

$$Q_i(c) := \int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \theta'_i \geq c} f(\boldsymbol{\theta}') d^n \boldsymbol{\theta}' = \int_c^1 \int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \theta'_i = x} f(\boldsymbol{\theta}') d^{n-1} \boldsymbol{\theta}' dx,$$

as the measure of agents of types not in E_i with valuations greater than c for i's product. Notice that $Q_i(c)$ is differentiable function in c.

Further note that $H_i(\hat{p}_i)$ is differentiable in $\hat{p}_i$. Since

$$\lim_{\hat{p}_i \downarrow 0} H_i(\hat{p}_i) = +\infty \qquad \lim_{\hat{p}_i \uparrow 1} H_i(\hat{p}_i) = 0,$$

there exists $\hat{c} \in (0,1)$ so that $H_i(\hat{c}) = Q_i(0)$. Both $H_i(\hat{p}_i)$ and $Q_i(c)$ are strictly decreasing. For $c \in (0,1)$, define:

$$\gamma(c) = H_i^{-1}(Q_i(c)).$$

Since $H_i(\cdot) \geq Q_i(\cdot)$, $\gamma(c) \geq c$. Also, $\gamma(c)$ is strictly increasing in c, and $\gamma(0) = \hat{c}$.

For each $\boldsymbol{\theta} \notin E_i$, if we raise θ_i to $\gamma(\theta_i)$, the resulting distribution of $\gamma(\theta_i)$ will be:

$$\int_{\substack{\boldsymbol{\theta}' \in V^n \setminus E_i : \\ \gamma(\theta'_i) \geq c}} f(\boldsymbol{\theta}') d^n \boldsymbol{\theta}' = \int_{\substack{\boldsymbol{\theta}' \in V^n \setminus E_i : \\ \theta'_i \geq Q_i^{-1}(H_i(c))}} f(\boldsymbol{\theta}') d^n \boldsymbol{\theta}' = Q_i(Q_i^{-1}(H_i(c))) = H_i(c)$$

Hence, if we indeed raised θ_i to $\gamma(\theta_i)$, the condition in the theorem will satisfied with equality for all $c \in (\hat{c}, 1)$.

Further note that by writing

$$H_i(c) = \int_{\substack{\boldsymbol{\theta}' \in V^n \setminus E_i : \\ \gamma(\theta'_i) \geq c}} f(\boldsymbol{\theta}') d^n \boldsymbol{\theta}' = \int_{\gamma^{-1}(c)}^1 \left(\int_{\substack{\boldsymbol{\theta}' \in V^n \setminus E_i : \\ \theta'_i = x}} f(\boldsymbol{\theta}') d^{n-1} \boldsymbol{\theta}' \right) dx$$

we have that

$$\frac{dH_i(c)}{dc} = -\int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \gamma(\theta'_i) = c} f(\boldsymbol{\theta}') d^{n-1} \boldsymbol{\theta}' \frac{d\gamma^{-1}(c)}{dc}. \tag{1}$$

For $\hat{p}_i < m_i$,

$$\frac{\partial G_i(\hat{p}_i, m_i)}{\partial \hat{p}_i} = -\frac{m_i}{\hat{p}_i^2} \int_{\boldsymbol{\theta}' \in E_i : \theta'_i = m_i} f(\boldsymbol{\theta}') d^{n-1} \boldsymbol{\theta}'$$

and

$$\frac{dH_i(\hat{p}_i)}{d\hat{p}_i} = \int_{m_i > \hat{p}_i} \frac{\partial G_i(\hat{p}_i, m_i)}{\partial \hat{p}_i} dm_i. \tag{2}$$

For all $\boldsymbol{\theta} \in E_i$, $x_i(\{\theta_i\}|\boldsymbol{\theta}) = 1$ fulfilling **separation**. For $\boldsymbol{\theta} \notin E_i, \theta_i \in (0,1)$, we construct the density

$$\psi_i(m_i|\boldsymbol{\theta}) := \begin{cases} \left(\frac{\partial G_i(\hat{p}_i, m_i)}{\partial \hat{p}_i} \Big/ \frac{dH_i(\hat{p}_i)}{d\hat{p}_i} \right) \Bigg|_{\hat{p}_i = \gamma(\theta_i)} & \text{if } m_i > \gamma(\theta_i); \\ 0 & \text{otherwise.} \end{cases}$$

Notice that above construction also fulfils **Consumer IC**, since $\gamma(\theta_i) \geq \theta_i$. It is also straightforward to verify that the $\psi_i(m_i|\boldsymbol{\theta})$ constructed above is a valid density: it is positive because both the denominator and numerator are strictly negative; it also integrates to one:

$$\int_0^1 \psi_i(m_i|\boldsymbol{\theta}) dm_i = \int_{\gamma(\theta_i)}^1 \psi_i(m_i|\boldsymbol{\theta}) dm_i = 1$$

from equation 2 above.

It remains to verify **Firm IC - sufficient**. For $m_i \in (0,1), \hat{p}_i < m_i$,

$$\begin{aligned} \int_{\boldsymbol{\theta} \in V^n \setminus E_i, \theta_i \geq \hat{p}_i} \psi_i(m_i|\boldsymbol{\theta}) f(\boldsymbol{\theta}) d^n \boldsymbol{\theta} &\leq \int_{\boldsymbol{\theta} \in V^n \setminus E_i, \gamma(\theta_i) \geq \hat{p}_i} \psi_i(m_i|\boldsymbol{\theta}) f(\boldsymbol{\theta}) d^n \boldsymbol{\theta} \\ &= \int_{\gamma(\theta_i) \geq \hat{p}_i} \psi_i(m_i|\boldsymbol{\theta}) \left(\int_{\substack{\boldsymbol{\theta}' \in V^n \setminus E_i: \\ \gamma(\theta'_i) = \gamma(\theta_i)}} f(\boldsymbol{\theta}') d^{n-1} \boldsymbol{\theta}' \right) \frac{d\theta_i}{d\gamma(\theta_i)} d\gamma(\theta_i) \\ &= \int_{\gamma(\theta_i) \geq \hat{p}_i} \psi_i(m_i|\boldsymbol{\theta}) (-1) \frac{dH_i(c)}{dc} \bigg|_{c = \gamma(\theta_i)} d\gamma(\theta_i) \\ &= \int_{\gamma(\theta_i) = \hat{p}_i}^{\gamma(\theta_i) = m_i} \frac{\partial G_i(c, m_i)}{\partial c} \bigg|_{c = \gamma(\theta_i)} (-1) d\gamma(\theta_i) \\ &= \int_{c = \hat{p}_i}^{c = m_i} \frac{\partial G_i(c, m_i)}{\partial c} (-1) dc \\ &= G_i(\hat{p}_i, m_i) \end{aligned}$$

where the first inequality is from $\gamma(\theta_i) \geq \theta_i$ and the second equality is from equation 1 above. Hence **Firm IC - sufficient** is fulfilled and by Lemma 2^{C}, $\boldsymbol{\Psi}^* \neq \emptyset$.

We now show the converse direction: suppose, towards a contradiction, that the condition in Theorem 1^{C} is not fulfilled, but $\boldsymbol{\Psi}^* \neq \emptyset$. By Lemma 2^{C}, there must exist some information structure ψ satisfying **Separability**, **Consumer IC**, and **Firm IC - necessary**.

Since the condition in Theorem 1^{C} does not hold, there must exist some $i \in \mathcal{N}$ and some $\hat{\theta}_i \in (0,1)$ such that

$$H_i(\hat{\theta}_i) < \int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \theta'_i \geq \hat{\theta}_i} f(\boldsymbol{\theta}') d\boldsymbol{\theta}'.$$

which in turn implies

$$\int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \theta'_i \geq \hat{\theta}_i} f(\boldsymbol{\theta}') d\boldsymbol{\theta}' > H_i(\hat{\theta}_i) = \int_{m_i \in M_i} G_i(\hat{\theta}_i, m_i) dm_i = \int_{m_i > \hat{\theta}_i} G_i(\hat{\theta}_i, m_i) dm_i.$$

But notice that we can write

$$\begin{aligned} \int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \theta'_i \geq \hat{\theta}_i} f(\boldsymbol{\theta}') d\boldsymbol{\theta}' &= \int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \theta'_i \geq \hat{\theta}_i} \Big(x_i([0, \hat{\theta}_i] | \boldsymbol{\theta}') + x_i((\hat{\theta}_i, 1] | \boldsymbol{\theta}') \Big) f(\boldsymbol{\theta}') d\boldsymbol{\theta}' \\ &= \int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \theta'_i \geq \hat{\theta}_i} x_i((\hat{\theta}_i, 1] | \boldsymbol{\theta}') f(\boldsymbol{\theta}') d\boldsymbol{\theta}' \end{aligned}$$

where the second equality is by **Consumer IC**. Hence, we have:

$$\int_{\boldsymbol{\theta}' \in V^n \setminus E_i : \theta'_i \geq \hat{\theta}_i} x_i((\hat{\theta}_i, 1] | \boldsymbol{\theta}') f(\boldsymbol{\theta}') d\boldsymbol{\theta}' > \int_{m_i > \hat{\theta}_i} G_i(\hat{\theta}_i, m_i) dm_i$$

but this violates **Firm IC - necessary**, a contradiction. □

B.2.2 Consumer-optimal design in the continuous case.

As before, we begin by characterizing a lower bound on the profits of firm $i \in \mathcal{N}$. For the same reason as in the main text, this is given by

$$\Pi_i^{C*}(\mathbf{0}) = \max_{p_i \in [0,1]} p_i \int_{\boldsymbol{\theta} \in V^n : \theta_i - \max_{j \neq i} \theta_j \geq p_i} f(\boldsymbol{\theta}') d\boldsymbol{\theta}'.$$

This is the lowest profit that i can make in any equilibrium induced by any information structure. Denote one of the optimal price of the above problem by p_i^*.

We are once again interested in the distribution of residual valuations—the maximum amount consumers are willing to pay for i's product given that they face prices 0 for all firms $j \neq i$— among consumers in E_i. Define i's effective demand function as

$$D_i^{\text{eff}}(\hat{p}_i) = \int_{\boldsymbol{\theta} \in E_i : \theta_i - \max_{j \neq i} \theta_j \geq p_i} f(\boldsymbol{\theta}') d\boldsymbol{\theta}' \quad \text{for all } \hat{p}_i \in [0,1]$$

which gives the total demand for i's product if for all consumers in E_i, i sets price $\hat{p}_i$ and all other firms $j \neq i$ set price 0. Observe that $D_i^{\text{eff}}(0) - D_i^{\text{eff}}$ is simply a renormalized right-continuous distribution function and so corresponds to a unique measure x_i^{eff} on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ which fulfils $x_i^{eff}([p, +\infty)) = D_i^{\text{eff}}(p)$ for all p.[25]

Now normalizing without loss so that $x_i^{eff}([0,1]) = 1$, Theorem 1B of BBM shows that for any distribution of residual valuations, there exists a $\sigma_i^{\text{eff}} \in \Delta\Delta[0,1]$ such that

$$\int_{x \in \Delta[0,1]} x(B) \sigma_i^{\text{eff}}(dx) = x_i^{\text{eff}}(B) \quad \text{for all Borel sets } B \in \mathcal{B}([0,1])$$

[25]Uniqueness follows from the $\pi - \lambda$ Theorem.

, which has **extremal and uniform profit preserving** property: for each distribution $y \in \text{supp}(\sigma_i^{\text{eff}})$,

$$p_i^* \in \text{supp}(y) = \underset{p \in [0,1]}{\text{argmax}}\, p * y(\text{supp}(y) \cap [p, 1]),$$

If the information designer further segment each E_i by σ_i^{eff}, and send each segment $y \in \text{supp}(\sigma_i^{\text{eff}})$ to all firms publicly, then firm i charge $\min \text{supp}(y)$ and other firms charge zero is an equilibrium. In that equilibrium, each firm i's profit is driven down to $\Pi_i^C(\mathbf{0})$. But since the allocation is also efficient, this achieves the upper bound of consumer surplus:

$$CS^C = S^C - \sum_{i=1}^{n} \Pi_i^C(\mathbf{0})$$

where

$$S^C = \sum_{i=1}^{n} \int_{\boldsymbol{\theta} \in E_i} f(\boldsymbol{\theta}) \theta_i d\boldsymbol{\theta}$$

is the total surplus available.

Theorem $2^{\mathbf{C}}$. The consumer-optimal information structure takes the following form: For each $i \in \mathcal{N}$, we apply a uniform profit preserving extremal segmentation according to the distribution of residual valuations and give this as public information.[26]

B.3 Distributions fulfilling Theorem $1^{\mathbf{C}}$. In this Online Appendix, we will focus on duopoly case with symmetric distribution of valuations.

B.3.1 Horizontal Differentiation: Uniform

We start by considering the case of perfectly anti-correlated distributions over $[0, 1]$, where a consumer of type $\theta \in [0, 1]$ has preference θ for firm 1's product, and $1 - \theta$ for firm 2's product. It is straightforward to check that the uniform distribution over $[0, 1]$ fulfils the condition in Theorem 1:

$$\begin{aligned}
H(\hat{\theta}) &= \int_{1/2}^{1} \underbrace{\frac{m - \hat{\theta}}{\hat{\theta}} f(m)}_{G(\hat{\theta}, m)} dm \\
&= \left[\frac{1}{2\hat{\theta}} m^2 - m \right]_{1/2}^{1} \\
&= \frac{3}{8\hat{\theta}} - 1/2 > F(\hat{\theta}) := \int_{\hat{\theta}}^{1/2} dm = 1/2 - \hat{\theta} \quad \text{for all } \hat{\theta} \in [0, 1/2].
\end{aligned}$$

B.3.2 Horizontal Differentiation: Normal

We now consider the truncated normal distribution over $[0, 1]$ with mean $1/2$ and variance σ^2. In particular, letting $X \sim N(1/2, \sigma^2)$. The truncated variable $\bar{X}$ is distributed as

[26] Explicitly, we can associate each segment with a unique message.

the conditional distribution of X conditional on $X \in [0,1]$. Hence $\bar{X}$ has density:

$$\hat{f}(m) = \begin{cases} \dfrac{\frac{1}{\sigma}\phi(\frac{m-1/2}{\sigma})}{S(\sigma)} & m \in [0,1] \\ 0 & m \notin [0,1] \end{cases}$$

where $S(\sigma) := \mathbf{P}[X \in [0,1]] = \Phi(\frac{1}{2\sigma}) - \Phi(\frac{-1}{2\sigma})$, and Φ and ϕ are the CDF and PDF of a standard normal random variable respectively. We will be interested in computing the ranges of σ^2 under which the condition of Theorem 1^{C} is fulfilled.

As a first observation, observe that as $\sigma \to \infty$, $\hat{f}(m) \to 1$ for all $m \in [0,1]$ which, we know from the calculations above, fulfils the condition in Theorem 1^{C}. We now turn to characterizing the values of σ under which the condition of Theorem 1^{C} remain fulfilled.

Remark 1. When valuations are perfectly anti-correlated and distributed as the truncated normal distribution with parameter σ, larger σ corresponds to more polarised distributions of valuations.

Proof. By symmetry, it suffices to show that $\int_\theta^{1/2} \hat{f}(m)dm$ decreases in σ for all $\theta \in [0,1/2]$ i.e., for all $\theta \in [1/2,1]$, the mass of consumers over E_2 who have preferences greater than θ for 1's product decreases. We show this through a direct calculation. Noting

$$\int_\theta^{1/2} \hat{f}(m)dm = \frac{\Phi(\frac{1/2-\theta}{\sigma}) - 1/2}{2[\Phi(\frac{1}{2\sigma}) - 1/2]}$$

and so

$$\frac{\partial}{\partial\sigma}\left(\int_\theta^{1/2} \hat{f}(m)dm\right) = \frac{A}{4\sigma^2[\Phi(\frac{1}{2\sigma}) - 1/2]^2}$$

where

$$A = (2\theta - 1)\phi\left(\frac{1/2-\theta}{\sigma}\right)\left[\Phi\left(\frac{1}{2\sigma}\right) - 1/2\right] + \phi\left(\frac{1}{2\sigma}\right)\left[\Phi\left(\frac{1/2-\theta}{\sigma}\right) - 1/2\right]$$

It remains to show $A \leq 0$ for all $\theta \in [0,1/2]$.

$$\frac{\partial A}{\partial \theta} = \phi\left(\frac{1/2-\theta}{\sigma}\right)B$$

$$B = \left(2\Phi\left(\frac{1}{2\sigma}\right) - 1\right)\left(1 - \frac{(1/2-\theta)^2}{\sigma^2}\right) - \frac{1}{\sigma}\phi\left(\frac{1}{2\sigma}\right)$$

where we used that $\phi'(x) = -x\phi(x)$ for all $x \in \mathbb{R}$. Now noticing that (i) B is strictly increasing in θ for $\theta \in [0,1/2]$; (ii) $\phi > 0$, and (iii) $A|_{\theta=0} = A|_{\theta=1/2} = 0$, we have the result. □

Now checking the condition in Theorem 1^{C},

$$
\begin{aligned}
H(\hat{\theta}) &= \int_{1/2}^{1} \left(\frac{m-\hat{\theta}}{\hat{\theta}}\hat{f}(m)\right)dm \\
&= \frac{1}{\hat{\theta}}\int_{1/2}^{1} m\hat{f}(m)dm - \int_{1/2}^{1}\hat{f}(m)dm \\
&= \frac{1}{\hat{\theta}}\int_{1/2}^{1} m\hat{f}(m)dm - 1/2 \\
&= \frac{1}{\hat{\theta}S(\sigma)}\int_{1/2}^{1} \frac{m}{\sigma}\phi\left(\frac{m-1/2}{\sigma}\right)dm - 1/2 \\
&= \frac{1}{\hat{\theta}S(\sigma)}\int_{0}^{1/2\sigma} \left(y+\frac{1}{2\sigma}\right)\phi(y)\sigma dy - 1/2 \quad [y=(m-1/2)/\sigma] \\
&= \frac{\sigma}{\hat{\theta}S(\sigma)}\int_{0}^{1/2\sigma} y\phi(y)dy + \frac{1}{2\hat{\theta}S(\sigma)}\int_{0}^{1/2\sigma}\phi(y)dy - 1/2 \\
&= -\frac{\sigma}{\hat{\theta}S(\sigma)}\int_{0}^{1/2\sigma} \phi'(y)dy + \frac{\Phi(\frac{1}{2\sigma})-\Phi(0)}{2\hat{\theta}S(\sigma)} - 1/2 \quad [x\phi(x)=-\phi'(x)] \\
&= \frac{\sigma}{\hat{\theta}S(\sigma)}\left(\phi(0)-\phi(\frac{1}{2\sigma})\right) + \frac{1}{4\hat{\theta}} - 1/2 \quad [\text{Fundamental Theorem of Calculus}]
\end{aligned}
$$

which we compare against

$$F(\hat{\theta}) = \int_{\hat{\theta}}^{1/2}\hat{f}(m)dm = S(\sigma)^{-1}\left(\Phi\left(\frac{1/2-\hat{\theta}}{\sigma}\right) - 1/2\right).$$

Then the condition in Theorem 1^{C} is fulfilled if and only if $H(\hat{\theta}) \geq F(\hat{\theta})$ for all $\hat{\theta} \in [0, 1/2]$ or, equivalently, if

$$\frac{\sigma}{\hat{\theta}}\left(\phi(0)-\phi(\frac{1}{2\sigma})\right) + (\frac{1}{4\hat{\theta}} - 1/2)S(\sigma) \geq \Phi\left(\frac{1/2-\hat{\theta}}{\sigma}\right) - 1/2 \quad \text{for all } \hat{\theta} \in [0, 1/2]. \tag{3}$$

Observe that

$$\lim_{\sigma\to 0}\frac{\sigma}{\hat{\theta}}\left(\phi(0)-\phi(\frac{1}{2\sigma})\right) + (\frac{1}{4\hat{\theta}} - 1/2)S(\sigma) = \frac{1}{4\hat{\theta}} - 1/2 < \lim_{\sigma\to 0}\Phi\left(\frac{1/2-\hat{\theta}}{\sigma}\right) - 1/2 = 1/2$$

whenever $\hat{\theta} > 1/4$, so the producer-optimal outcome is never attainable since there is virtually no variation in preferences. Conversely, as $\sigma \to \infty$, both sides are zero in the limit and the condition is trivially fulfilled.

Now notice that the above remark implies that there is some $\bar{\sigma} > 0$ such that for all $\sigma \geq \bar{\sigma}$, the condition in Theorem 1^{C} is fulfilled; conversely, for $\sigma < \bar{\sigma}$, the condition is not fulfilled. We now turn to solving for $\bar{\sigma}$.

Rewriting Equation 3,

$$\sigma\left[\phi(0) - \phi\left(\frac{1}{2\sigma}\right)\right] + \frac{\Phi(\frac{1}{2\sigma}) - 1/2}{2} \geq C(\hat{\theta}, \sigma)$$

where

$$C(\hat{\theta},\sigma) = \hat{\theta}\left[\Phi\left(\frac{1/2-\hat{\theta}}{\sigma}\right) + \Phi\left(\frac{1}{2\sigma}\right) - 1\right]$$

$$\frac{\partial C(\hat{\theta},\sigma)}{\partial \hat{\theta}} = \Phi\left(\frac{1/2-\hat{\theta}}{\sigma}\right) + \Phi\left(\frac{1}{2\sigma}\right) - 1 - \frac{\hat{\theta}}{\sigma}\phi\left(\frac{1/2-\hat{\theta}}{\sigma}\right)$$

which is strictly decreasing in $\hat{\theta}$ for $\hat{\theta} \in [0, 1/2]$. Further noting $\frac{\partial C(\hat{\theta},\sigma)}{\partial \hat{\theta}}|_{\hat{\theta}=0} = 2\Phi(\frac{1}{2\sigma}) - 1 > 0$ and $\frac{\partial C(\hat{\theta},\sigma)}{\partial \hat{\theta}}|_{\hat{\theta}=1/2} = \Phi(\frac{1}{2\sigma}) - \Phi(0) - \frac{1}{2\sigma}\phi(0) < 0$, $C(\hat{\theta},\sigma)$ is maximized at the unique root of $\frac{\partial C(\hat{\theta},\sigma)}{\partial \hat{\theta}} = 0$.

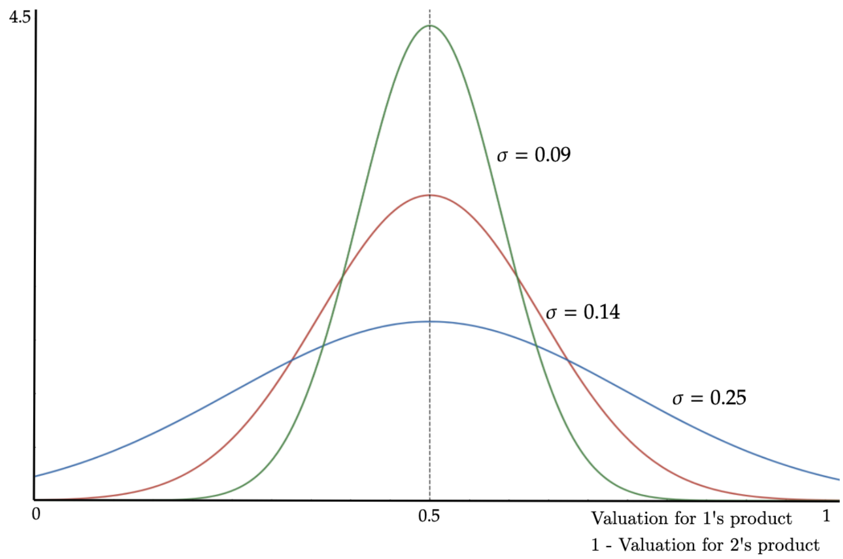
Hence, $\bar{\sigma}$ is characterized by the equations:

$$\bar{\sigma}[\phi(0) - \phi(\frac{1}{2\bar{\sigma}})] + \frac{\Phi(\frac{1}{2\bar{\sigma}}) - 1/2}{2} = \theta^*[\Phi\left(\frac{1/2-\theta^*}{\bar{\sigma}}\right) + \Phi(\frac{1}{2\bar{\sigma}}) - 1]$$

$$\Phi\left(\frac{1/2-\theta^*}{\bar{\sigma}}\right) + \Phi(\frac{1}{2\bar{\sigma}}) - 1 - \frac{\theta^*}{\bar{\sigma}}\phi\left(\frac{1/2-\theta^*}{\bar{\sigma}}\right) = 0.$$

Solving for the smallest $\bar{\sigma}$, we have $\bar{\sigma} \simeq 0.14$. The corresponding PDF of this distribution is shown in Figure 5 below (red line). For all $\sigma \geq \bar{\sigma}$, Theorem 1^C is fulfilled (e.g., the blue line); for all $\sigma < \bar{\sigma}$, Theorem 1^C is not (e.g., the green line).

Figure 5: Illustration of the truncated normal distribution



B.3.3 Bivariate Uniform

We now depart from the perfectly anti-correlated case and study how the condition in Theorem 1^C interacts with the degree of variation in consumer preferences. To do so, we study the uniform distribution over $[a, b]^2$ with $a = 1/2 - \delta$, $b = 1/2 + \delta$, for $\delta \in (0, 1/2]$. In particular, for any $(\theta_1, \theta_2) \in [a, b]^2$, $f(\theta_1, \theta_2) = 1/4\delta^2$.

Remark 2. Since the model is invariant to preference rescaling, this is equivalent to a distribution with uniform preferences over

$$\left[\frac{1/2-\delta}{1/2+\delta}, 1\right]^2$$

where δ controls the degree of variation in preferences: when $\delta \to 0$, we approach the dirac delta on the point $(1,1)$; when $\delta = 1/2$, we obtain the uniform distribution will full support over $[0,1]^2$.

It turns out that in this setting, the condition in Theorem 1^{C} is fulfilled if and only if $\delta = 1/2$. By direct calculation in Mathematica,

$$\begin{aligned}
&H(\hat{\theta}) - F(\hat{\theta}) = \frac{(10\delta - 4\hat{\theta} - 1)(2\delta - 2\hat{\theta} + 1)^2}{129\delta^2\hat{\theta}} \geq 0 \quad \text{for all } \hat{\theta} \in [a,b] \\
\iff &\min_{\hat{\theta}\in[a,b]} 10\delta - 4\hat{\theta} - 1 \geq 0 \\
\iff &\delta \geq 1/2.
\end{aligned}$$

This implies that in the duopoly case with uniform and uncorrelated preferences, the intermediary is just able to structure information to achieve the producer-optimal outcome.